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ROYAL AERONAUTICAL SOCIETY

(FOUNDED 1897 in succession to the ANNUAL REPORTS)

Edited for the Council by J. LAURENCE PRITCHARD, Fellow

No. 162 JUNE, 1924 VOL. XXVIII

NOTICES

Election of Members

The following Members were elected at a meeting of the Council held on May 13th :—

Associate Fellows.—Mr. R. O. King, Mr. C. H. L. Needham, and Mr. N. S. Norway.

Silver Medal

The Council have awarded the Society's Silver Medal for the year 1923 to Squadron Leader R. M. Hill, M.C., A.F.C., A.F.R.Ae.S., R.A.F., for his paper on "The Manœuvres of Inverted Flight," published in the Journal in December, 1923.

Busk Studentship

A Studentship in Aeronautics to the value of £150 per annum has been established at Cambridge University in memory of Edward Teshmaker Busk, who in 1914 lost his life while flying the first stable aeroplane. Full details and forms of application can be obtained from Professor B. M. Jones, Engineering Laboratory, Cambridge.

Edward Busk Memorial Lecture Fund

With the consent of the Trustees, the Council have decided to utilise the income of this Fund, which has been accumulating since the raising of the Fund in 1914, in awarding a prize annually for the best paper received on some subject of a technical nature in connection with Aeroplanes (including Seaplanes). Detailed regulations will be found on the next page.

Library

Members are reminded that Fellows, Associate Fellows, Students and full Members are entitled to borrow books from the Library, subject to the payment of carriage both ways. Other grades are entitled to consult books in the Library, but not to borrow them.

Vacancy

Attention is drawn to the following advertisement which has recently appeared :—"TEST ASSISTANT required for work on aeroplanes' airworthiness calculations. Applicants must have technical training to the standard B.Sc. degree and works and drawing office experience. Salary about 60s. per week plus Civil Service bonus, giving a present total of about 97s. Ex-Service man preferred. Royal Aircraft Establishment, South Farnborough, Hants."

W. LOCKWOOD MARSH, *Secretary.*

EDWARD BUSK MEMORIAL PRIZE

Regulations

From the income of the above Fund, a sum of Twenty Guineas will be offered as a prize for the best paper received by the Royal Aeronautical Society on some subject of a technical nature in connection with Aeroplanes (including Seaplanes).

The prize is open to international competition. The Royal Aeronautical Society retains the right to withhold the prize in any year if it is considered that no paper is of sufficient merit to justify an award.

Intending competitors should send their names to the Secretary of the Royal Aeronautical Society, 7, ALBEMARLE STREET, LONDON, W.1, on or before September 30th, 1924, with such information in regard to the projected scope of their papers as will enable arrangements to be made for their examination. The closing date for the receipt of papers will be December 31st, 1924.

Papers, which must be submitted in either French or English, should in all cases be typed, and a copy should be retained by the Author, as the Society can take no responsibility for the loss of copies submitted to it.

Successful papers will become the absolute property of the Society, and will in most instances be published in the JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY. A signed undertaking must accompany each paper to the effect that publication has not already taken place and that the Author will not communicate it elsewhere until the Society's award is published.

The Society attaches special importance to papers showing original work, and due acknowledgment must be made by the Author of the source of any special information.

R38 MEMORIAL PRIZE, 1923.

The Council have specially commended the following paper, submitted for the R.38 Memorial Prize, 1923, on "The Strength of Rigid Airships," by Mr. C. P. Burgess, Commander J. C. Hunsaker, U.S.N., Hon. F.R.Ae.S., and Mr. Starr Truscott.

THE STRENGTH OF RIGID AIRSHIPS

BY C. P. BURGESS, J. C. HUNSAKER AND S. TRUSCOTT.

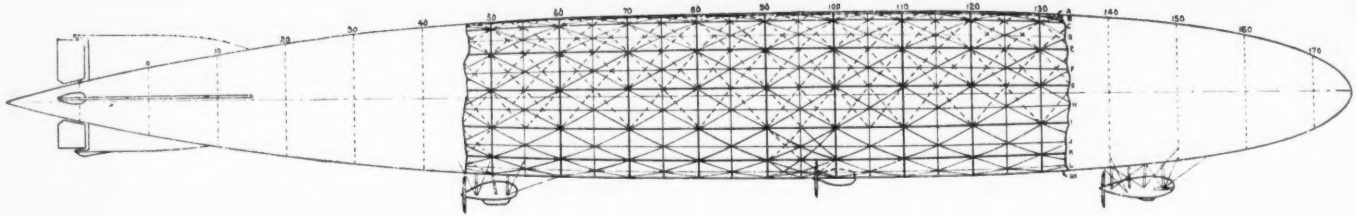
Introduction

The rigid airship in its modern form is a German conception, but the nature of the strength calculations employed by the Germans has been a carefully guarded secret. When the U.S. Navy undertook the construction of rigid airships it was arranged to have one ship designed and built in the United States, the ZR.1, and one ship designed and built in England, R.38 (ZR.2). Lacking practical information and experience, fundamental theory had to be resorted to in undertaking the design of ZR.1, and all available information had to be analysed, compared and, so far as possible, tested. After the unfortunate loss of R.38 the calculations for ZR.1 were reviewed and additional calculations and investigations undertaken.

The authors of this paper, under the direction of Rear-Admiral W. A. Moffett, Chief of the Bureau of Aeronautics, U.S. Navy Department, have been charged with the design of ZR.1 and have had in the course of duty to make, or cause to be made, such studies and investigations as would assist in analysing and simplifying the problem as much as was possible in advance of full-scale experiments in flight. Believing in the widest possible dissemination of scientific and technical information with regard to aeronautics with a view to advancing the art, Admiral Moffett has authorised the submission of this paper to the Council of the Royal Aeronautical Society under the R.38 Memorial Research Fund Competition.

The work for the bureau has necessarily included a vast amount of detailed test and investigation of materials, including light alloys, helium gas, goldbeater's skin substitutes, protective coatings and textiles, as well as investigations concerning many of the details of rigid airship construction, fabrication of joints, rivets, wire terminals, bow mooring gear, rudders, etc. These items, while important to a harmonious design, are not to be considered fundamental and will not be treated here. Also for brevity, strength calculations for fins and for the keel, due to concentrated weights, are omitted.

The object of the authors is to present in as compact and yet complete a form as is possible within the limits of a professional paper the present state of our knowledge with regard to the general strength of rigid airships. To do this no attempt will be made to be historically accurate in giving credit to those who may have originated certain elements of the theory, as it is assumed that the actual priority will be found to belong to some German who, for reasons of State, was not permitted to publish the results of his work. However, where publications of others have been used for the purpose of comparison or adaptation credit will be given. In particular, the authors must acknowledge the valuable contributions which have been made by Professor Hovgaard, Dr. Munk and Mr. Pagon, whose theoretical analyses of different phases of the general problem, made at the instigation of the Bureau of Aeronautics, have served to close gaps in our knowledge and to permit a practical calculation of structural strength to be made with the assurance that the magnitude of the errors involved can be estimated and are not greater than are usually tolerated in engineering work.



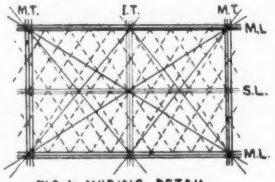


FIG. 1a. WIRING DETAIL.

M.T. MAIN TRANSVERSE
I.T. INTERMEDIATE TRANSVERSE
M.L. MAIN LONGITUDINAL
S.L. SECONDARY LONGITUDINAL
——— MAIN SHEAR WIRE
- - - - - GAS CELL NETTING
——— SECONDARY SHEAR WIRE

SCALE OF PANEL

SCALE OF SHIP

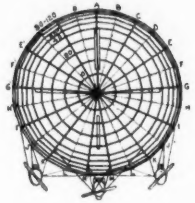


FIG. 1

TYPICAL RIGID AIRSHIP - GENERAL ARRANGEMENT OF LONGITUDINALS, TRANSVERSES AND WIRING.

The complete strength calculations for a rigid airship are extremely voluminous, even when compared with the calculations required for a warship, and no attempt will be made to go farther than is necessary in order to present in a rational manner the general methods which have been devised by us, or extended and adapted from the work of others. When necessary the discussion of novel formulæ and sample calculations will be given in appendices.

It is believed that a complete discussion of the strength of rigid airships has never before been published. This paper, if complete as to scope, cannot pretend to be final nor to include those practical factors of experience known as "factors of safety," which form the sure foundation of engineering practice. This information it is planned to obtain later by means of strain measurements on the actual ship in the shed under known static forces and similar strain measurements in the air under the unknown aerodynamic forces. At the present time the necessary factors of safety have been obtained by comparison with the known structure of several successful German airships and with R.38. The results are consistent and give internal evidence of a close approximation to the truth.

The problem of the longitudinal strength of rigid airships is especially difficult, both in respect to the determination of the aerodynamic loads which may be imposed upon the airship, and the distribution of stresses from known bending moments and shearing forces. No serious attempt to determine the aerodynamic forces appears to have been published until after the loss of R.38, although the authors of this paper had, in 1920, investigated the effect of sudden application of the rudders.

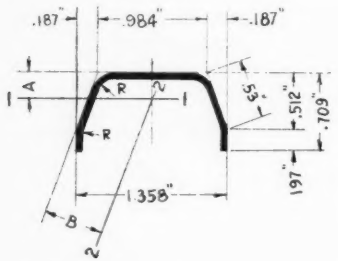
As a result of the loss of the R.38, great attention has recently been given to the study of the aerodynamic forces upon airships. The National Physical Laboratory led the way in pressure plotting experiments upon a model in the wind tunnel; but this method gives no adequate theory of the distribution of transverse air force for general use, and cannot be applied to an airship turning, except by the aid of some rather doubtful extrapolation from results at a fixed angle of yaw. At the instigation of the Bureau of Aeronautics, Dr. M. M. Munk, of the Staff of the National Advisory Committee for Aeronautics, undertook a theoretical investigation of the motion of bodies of revolution, such as an airship, in a perfect fluid. He obtained approximate solutions for the cases of flight at a fixed angle of yaw or pitch, and for steady turning. Although the behaviour of a perfect fluid differs considerably from that of real fluid, such as air, comparison with the N.P.L. results for a fixed angle of yaw, German dynamic lift trials, and British turning trials, indicates that Dr. Munk's theory is sufficiently accurate for practical computations, while offering the enormous practical convenience of general applicability for direct computation.

The calculation of the stresses resulting from known primary bending moments is also a unique problem. Owing to the large number of longitudinal girders and the limberness of rigid airships in shear, the application of the ordinary bending theory (beam theory) to the primary stresses gives very inexact results. An exact solution of the problem by the method of least work, or by deflections, seems to be wholly impracticable because of the great number of simultaneous equations involved. Here, as in the problem of the aerodynamic forces, the authors have striven for practical methods of calculation, accepting a reasonable but definite margin of error. Means for obtaining such results are developed in this paper.

The transverse strength has hitherto received very inadequate treatment. Even engineers familiar with strength calculations for airships have often proceeded upon false assumptions regarding the tendency of the intermediate frames to distort. This matter will be discussed in the light of the full-scale tests conducted by the authors on the hull of ZR.1 while building.

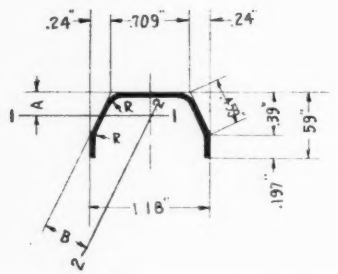
XIIM

A considerable, although fragmentary, body of literature on the strength of rigid airships has already been published (see bibliography and sources of information in Appendix I.); but it is believed there has not been hitherto available a comprehensive description of adequate methods for computing the general longitudinal and transverse strength. A knowledge of the general features of modern rigid airships is presupposed. The reader unfamiliar with the subject is referred to the bibliography. For descriptions of rigid airships, two papers by Commander Garland Fulton (C.C.), U.S.N., published in the Proceedings of the U.S. Naval Institute in October and November, 1921, are especially recommended.



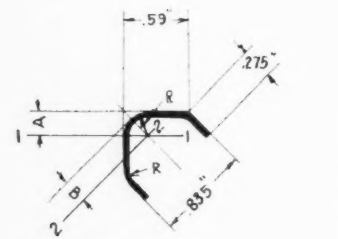
	A1	A2	A3
THICKNESS-INS.	.039	047	055
SECT. AREA - SQ. INS.	.09306	.11109	.12892
R (INSIDE RADII)-INS.	.120	.160	.160
A. (FROM TOP)-INS.	.230	.231	.232
B. (FROM WEB)-INS.	.541	.542	.543
I. (ABOUT 1-1)-INS^4	.00478	.00572	.00659
I. (ABOUT 2-2)-INS^4	.02064	.02433	.02763

CHANNELS A1, A2 & A3.



	B1	B2	B3	B4	B5	B6
THICKNESS-INS.	.031	039	.047	.055	.063	.079
SECT AREA-SQ.INS	.06121	.07604	.09066	.10495	.11933	.14457
R (INSIDE RADII)-INS	.120	.120	.140	.160	.175	.175
A (FROM TOP)-INS	.2070	.2088	.2106	.2124	.2142	.2160
B (FROM WEB)-INS	.4130	.4138	.4146	.4154	.4162	.4170
I (ABOUT 1-1)-INS^4	.00218	.0029	.00313	.00357	.00386	.00410
I (ABOUT 2-2)-INS^4	.00830	.0103	.01242	.01425	.01603	.01908

CHANNELS B1, B2, B3, B4, B5 & B6



	N1	N2	N3	N4
THICKNESS-INS	.039	047	.055	.063
SECT AREA-SQ.INS.	.03364	.01545	.08735	.09904
R (INSIDE RADII)-INS.	.120	.160	.160	.175
A (FROM WEB)-INS.	.238	.242	.246	.250
B (FROM TOP)-INS	.275	.261	.260	.256
I (ABOUT 1-1)-INS^4	.00317	.00432	.00497	.00555
I (ABOUT 2-2)-INS^4	.00214	.00230	.00257	.00290

ANGLES N1, N2, N3 & N4

FIG. 2.

Dimensions and properties of channels and angles.

Division of the Problem

The investigation into the strength of rigid airships may conveniently be separated into divisions as follows:—

Section I.—Strength of girders.
,, II.—Static forces affecting longitudinal strength.
,, III.—Aerodynamic forces affecting longitudinal strength.
,, IV.—Longitudinal strength.
,, V.—Secondary stresses resulting from the primary shearing forces.
,, VI.—Effect of gas pressure.
,, VII.—Effect of the outer cover.
,, VIII.—Transverse strength.
,, IX.—Factors of safety.
,, X.—Summary.

SECTION I.

Strength of Girders

The Girders

The type of girder developed by the Zeppelin Company for carrying the compressive and combined compressive and bending loads in rigid airships is believed to be the best to date. It has a triangular section of varying proportions according to the function. A channel is at each vertex of the triangle and the sides are laced with the characteristic stamped lattices designed to give a maximum of stiffness. The type has been frequently described and is illustrated in Figs. 3 to 6 and 10 to 14 inclusive.

At the present time duralumin is the only material which can be seriously considered for use in the making of these girders. At some future period other light alloys may be developed, or special steels, for ships of very large size, but these materials require no consideration here. What must be considered are the statements, sometimes met with, that duralumin cannot be made of uniform quality and properties, that it breaks down under alternating stresses, corrodes easily and is generally unreliable.

These statements can best be answered by reference to actual experience with the duralumin supplied for the construction of the ZR.1. This was supplied by two manufacturers and the results from tests of the product of each were equally good.

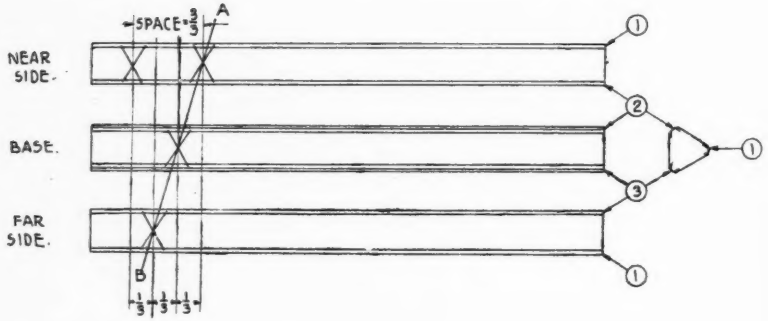
The Metal

The duralumin for the ZR.1 was specified to have the following chemical composition and physical properties:—

Copper	3·5 to 4·5%
Magnesium	·2 to ·75%
Manganese	·4 to 1.00%
Aluminium (minimum) ...	... 92.00%
Tensile strength at ultimate ...	... 55,000 lbs. per sq. in.
Tensile strength at elastic limit	... 25,000 lbs. per sq. in.
Elongation in 2in. at ultimate (test specimen ½in. wide) ...	... 18%
Specific gravity, not more than	... 2.85

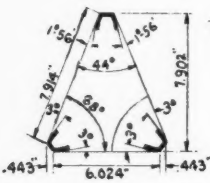
The following is the mean composition as determined from several hundred analyses of specimens from one contractor. Once in production this composition was held to very closely.

Copper	4.23%
Magnesium	·54%
Manganese	·52%
Iron	·45%
Silicon	.26%
Aluminum	94.00%



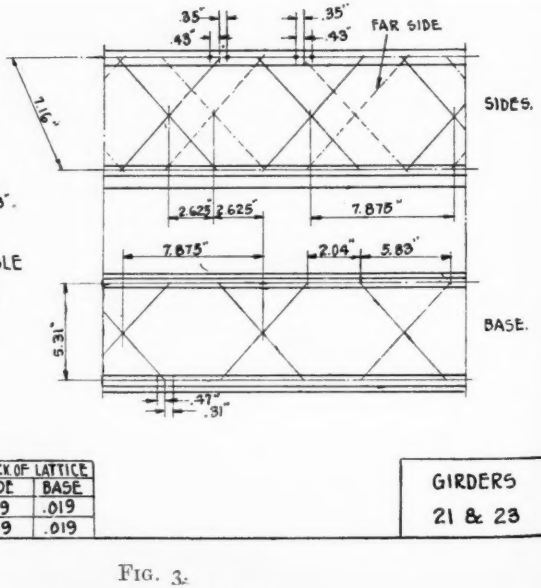
LINE A–B INDICATES THE DIRECTION OF STANDARD SPIRAL.
THE DIRECTION OF THE LINE OF SPIRAL IS ALWAYS N.EAST AND S.WEST.

SHOWING SPIRAL OF LATTICE CENTERS AROUND GIRDER.



GIRDER NO. 23 HAS DOUBLE LATTICE ON SIDES AT PITCH OF ½ OF 7.875"

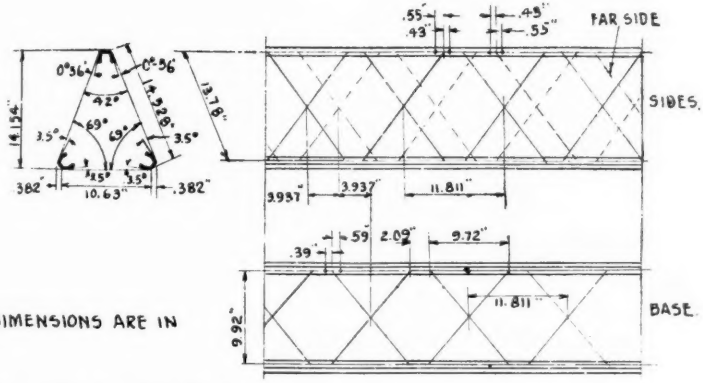
ALL DIMENSIONS ARE IN INCHES.



GIRDER NO.	APEX CHAN.	THICK.	BASE CHAN.	THICK.	THICK OF LATTICE SIDE	THICK OF LATTICE BASE
21	A1	.039	B1	.031	.019	.019
23	A1	.039	B1	.031	.019	.019

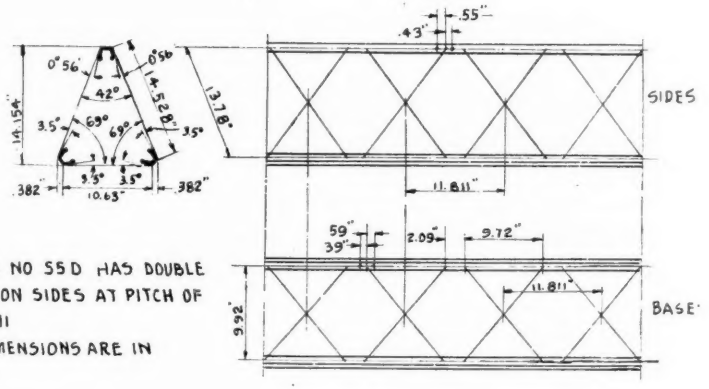
GIRDERS 21 & 23

FIG. 3.
Dimensions of girders.



GIRDER NO.	APEX CHAN.	THICK.	BASE CHAN.	THICK	THICK OF LATTICE SIDE	THICK OF LATTICE BASE
52	A1	.039	B2	.039	.019	.019
53	A2	.047	B2	.039	.019	.019
54	A2	.047	B3	.047	.019	.019
55	A3	.055	B3	.047	.023	.019
55A	A3	.055	B3	.047	.023	.019

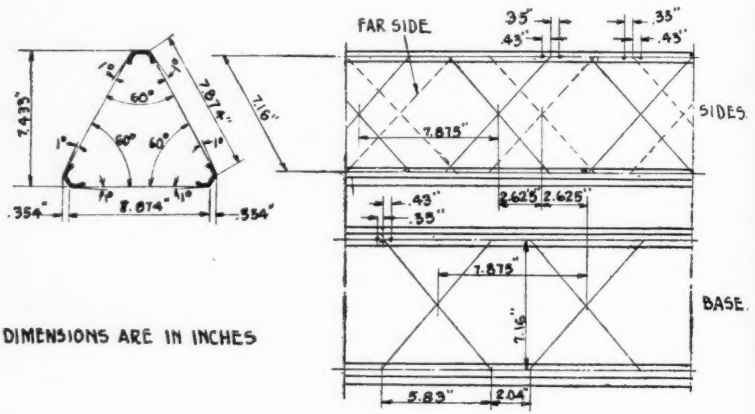
GIRDERS
52, 53, 54, 55 & 55A



GIRDER NO.	APEX CHAN.	THICK.	BASE CHAN.	THICK	THICK OF LATTICE SIDE	THICK OF LATTICE BASE
55B	A3	.055	B3	.047	.023	.019
55C	A3	.055	B3	.047	.019	.019
55D	A3	.055	B3	.047	.019	.019

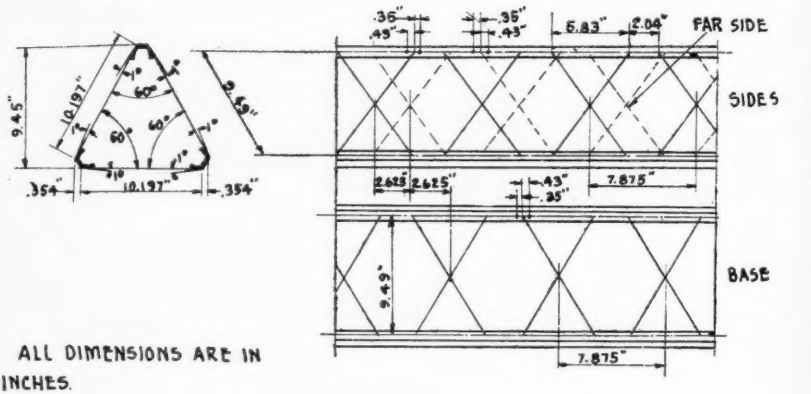
GIRDERS
55B, 55C & 55D

FIG. 4.
Dimensions of girders.



GIRDER NO.	APEX CHAN.	THICK.	BASE CHAN.	THICK.	THICK. OF LATTICE SIDE	THICK. OF LATTICE BASE
31B	B2	.039	B2	.039	.019	.019
32	B3	.047	B2	.039	.023	.023

GIRDERS 31B & 32



GIRDER NO.	APEX CHAN.	THICK.	BASE CHAN.	THICK.	THICK. OF LATTICE SIDE	THICK. OF LATTICE BASE
42	B3	.047	B3	.047	.019	.019

GIRDER 42

FIG. 5.
Dimensions of girders.

The results of the physical tests of 1,333 specimens are tabulated below:—

Ultimate tensile strength, lbs. per sq. in.	No. of samples.	Per cent. of total.
Below 39,999	40	3.0
40,000 to 44,999	4	0.3
45,000 to 49,999	24	1.8
50,000 to 54,999	78	5.8
55,000 to 59,999	970	72.8
Above 60,000	217	16.3
	1333	100.00
Elongation % in 2in.		
Below 15.99	69	5.1
16.00 to 16.99	28	2.1
17.00 to 17.99	47	3.5
18.00 to 18.99	455	34.2
19.00 to 19.99	317	23.8
20.00 to 20.99	252	18.9
21.00 to 21.99	82	6.2
Above 22.00	83	6.2
	1333	100.00

The tensile strength at the elastic limit is equally satisfactory but averages so high that it might be specified as 35,000lbs. per sq. in. without many rejections.

The specific gravity varied from 2.78 to 2.81 and may be safely taken as 2.80.

An extensive series of tests is being made for the Bureau of Aeronautics by the Bureau of Standards to determine the fatigue properties in bending of the sheet duralumin used on the ZR.1. This work is still incomplete, but results so far show great uniformity. An "endurance limit" in bending, that is to say, a fibre stress below which the number of reversals may be indefinitely increased without failure, has not been found, although very high numbers of reversals have been recorded. The latest reports on three typical specimens are given below.

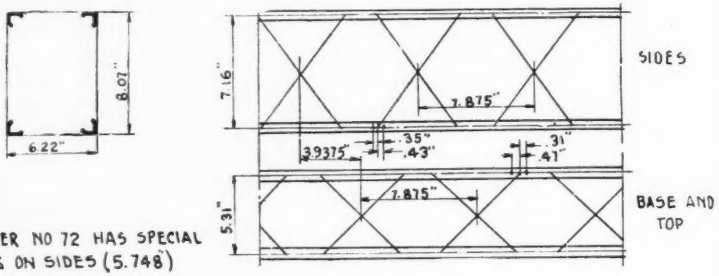
Thickness of sheet, inches.	Stress, lbs./in.^{2}	Number of reversals withstood without failure.
0.072	20,000	150,000,000
0.104	17,500	150,000,000
0.120	14,600	180,000,000

If the results to date are plotted with the results from rotating beam fatigue tests reported by other investigators it is found that the rotating beam specimens fail at the same number of alternations for a given stress as the plate specimens. When it is considered that the results are from material of different origin, it is believed that confidence in the excellent fatigue resistance of the metal is justified.

The resistance to corrosion of duralumin is excellent. If improperly heat treated or overworked it may corrode rapidly, but the same causes affect its physical properties, making it unfit for use in a structure.

Short lengths of the experimental girders, constructed for study in connection with the design of ZR.1, were suspended at three different places on the Atlantic coast, representing the north, middle and southern sections. At each point three lengths were used; one was suspended under a dock, over but not in contact with the water; the second was hung under a shelter, but not completely enclosed, near the water; while the third was hung up in the open and entirely exposed to the weather. The specimens were suspended by insulated wires and were left in place for a year.

The samples under the docks were coated with a black powdery coating which, while it dusted off, did not come away easily. Small white spots, scattered over the girder, were found to have only the slightest pits under them. The surface injury done by the year's exposure was really very slight. The samples exposed under the shelter showed the same darkening of the surface and white spots, but to a much less degree than those under the docks. Apparently the atmosphere

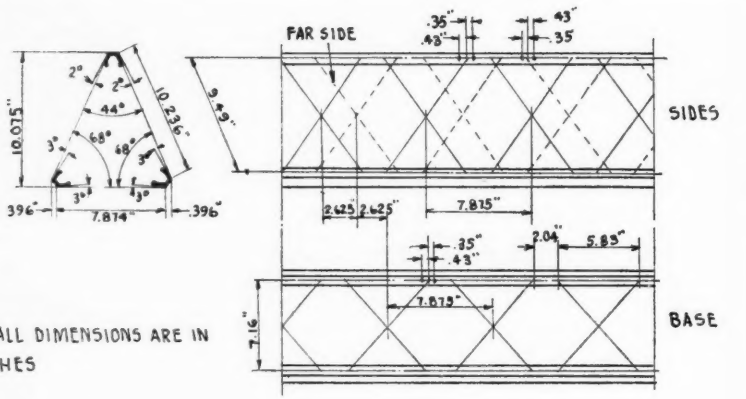


GIRDER NO 72 HAS SPECIAL SPACING ON SIDES (5.748")

ALL DIMENSIONS ARE IN INCHES.

GIRDER NO.	ANGLES	THICK.	THICK OF LATTICE SIDE	THICK OF LATTICE BASE
71	N3	.055	.019	.019
72	N3	.053	.019	.019

GIRDERS 71 & 72



ALL DIMENSIONS ARE IN INCHES

GIRDER NO.	APEX CHAN	THICK.	BASE CHAN.	THICK.	THICK OF LATTICE SIDE	THICK OF LATTICE BASE.
91	A1	.039	B1	.031	.019	.019
92	A2	.047	B2	.039	.019	.019

GIRDERS 91 & 92

FIG. 6.

Dimensions of girders.

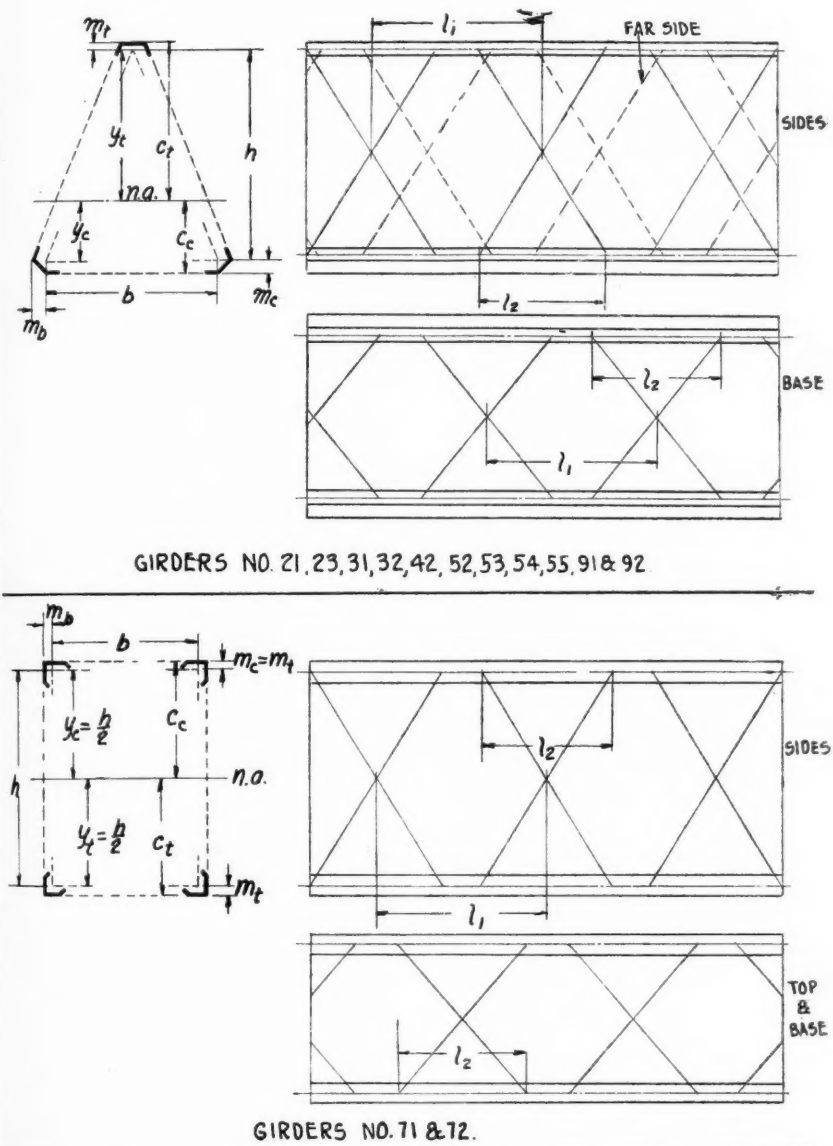


FIG. 7.

To illustrate properties of girders given in Table 1 and derivation of l_1 and l_2 of Table 6.

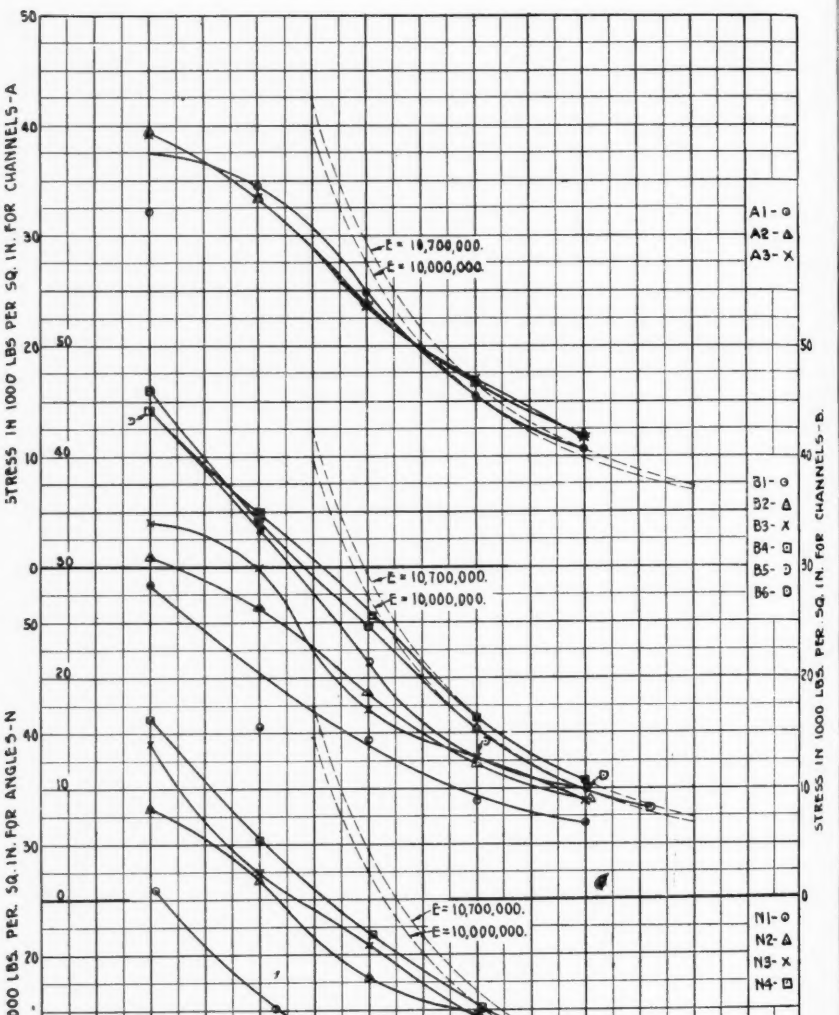


FIG. 8.

Column strength of channels and angles used as girder elements.

had been less saturated with salt water vapour. The samples exposed without shelter had taken on a brownish colour, which was apparently the blackish deposit just beginning to form in quantity. Through this deposit could still be traced the roll marks and other defects originally seen on the material. It was suggested that the rains had washed the corrosion products away as fast as they formed, but no evidence of active corrosion or deep pitting could be found.

Other tests have been made both in the open and in testing boxes. The results show plainly that, compared with steel, duralumin is very resistant to corrosion. The powdery products of corrosion are, however, very disagreeable to handle and obscure the surface. For these reasons it is preferred to coat the girders with some transparent medium which will check the corrosion and still permit of observing the condition of the material. A thin coat of spar varnish, applied hot by the spray gun, has been found to give good protection and to be simple in application. This coating is required more to ensure continued ability to examine the condition of the structure than to protect against corrosion.

Duralumin can be supplied in the form of sheet of very uniform thickness. The tolerances for the thickness of plate supplied for the ZR.1 were originally as follows:—

All dimensions in inches.

Nominal thickness.	Plus tolerance.	Minus tolerance.
.015 to .031	.001	.001
.039 to .079	.003	.003
.087	.005	.005

Once deliveries were well under way it was found that almost all of the sheet was being made on the thicker side of the nominal thickness and usually just under the maximum allowable thickness. This meant a substantial increase in the weight of the ship over estimates.

This over thickness was the result of the fear on the part of the roller that one more pass through the rolls, instead of bringing the thickness to the nominal, might bring it below the minimum and so cause the sheet to be rejected. This was remedied by the adoption of the following table of tolerances:—

Thickness in Inches.			%	%	%	Use in	
Nominal.	Min.	Max.	Variation.	Minus.	Plus.	Channels and Angles.	Girders.
0.015	.013	.016	20	13.3	6.7		
.019	.017	.020	15.8	10.5	5.3		
.023	.021	.024	13.0	8.7	4.3		
.027	.025	.028	11.1	7.4	3.7		
.031	.029	.032	9.7	6.5	3.2	B-1	21, 23, 91.
.039	.035	.041	15.4	10.3	5.1	A-1, B-2, N-1	21, 23, 52, 53, 91, 92, 31B, 32.
.047	.043	.049	12.8	8.5	4.3	A-2, B-3, N-2	42, 53, 54, 55, 55A-D, 92, 32.
.055	.051	.057	11.0	7.4	3.6	A-3, B-4, N-3	55A-D, 71, 72.
.059	.055	.061	10.2	6.6	3.4		
.063	.059	.065	9.5	6.3	3.2	B-5, N-4	
.071	.067	.073	8.4	5.6	2.8		
.079	.075	.081	7.6	5.1	2.5	B-6	
.087	.080	.090	11.5	7.7	3.8		

The "range" of tolerance has been shifted so that in endeavouring to come just under the maximum the roller is practically compelled to come to the nominal thickness. All the sheet for the ZR.1 was made to these tolerances, which have been found to be practical in manufacturing and to ensure the delivery of sheets averaging close to the nominal.

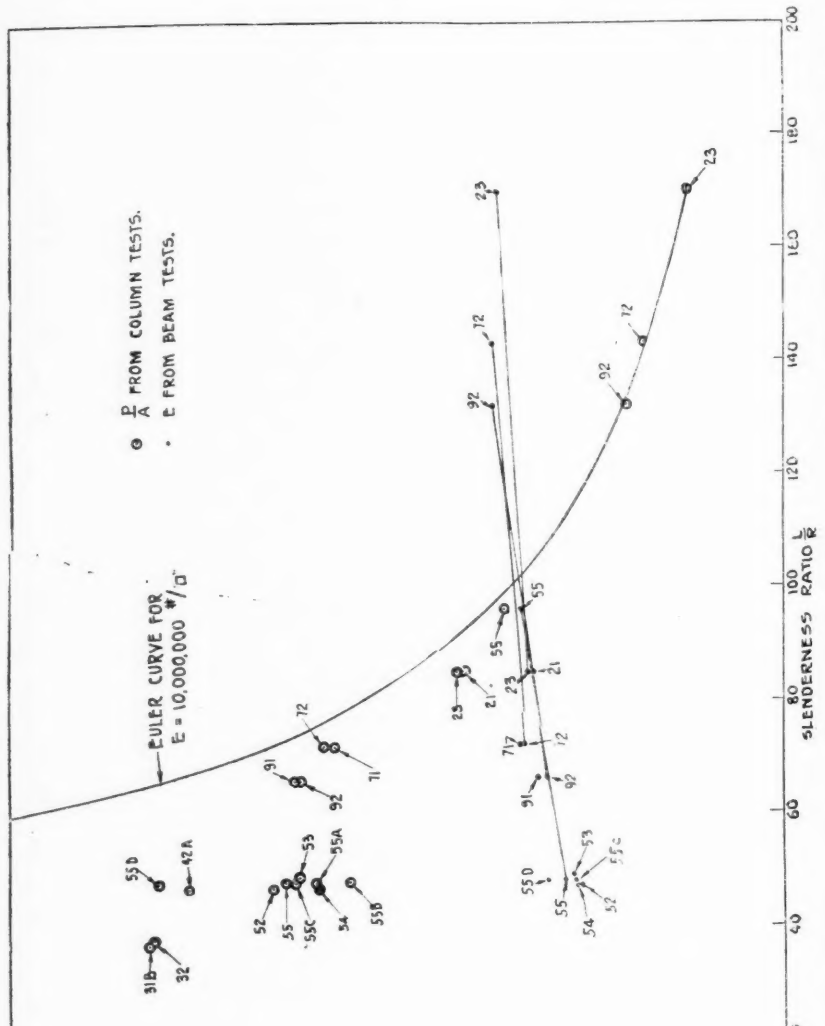


FIG. 9.

Column strength of girders and values of E from beam tests.

By the use of these tolerances it is possible that sheet and channels or angles may be from 5 to 10 per cent. under thickness. Of the sections used in the principal girders, as listed above, it is only the A-1, B2 and N-1 which can be expected to have the maximum variation. Assuming all three channels in the type 52 girder (see Figs. 2 and 4) to be of the minimum thickness, the sectional area is reduced from the nominal of .2395 sq. in. to .2148 sq. in. Assuming the girder of nominal thickness channels to carry the maximum transverse load reported in the beam tests (52b, Table 3), 1,733 lbs., the load carried with the same stress, but the lighter channels would be 1,560 lbs., which is to be compared with 1,588 lbs., the loads reported for beams 52a and 52c.

It is most improbable that all three channels in one girder would be of the minimum thickness. It was observed, however, that the girders always failed in the thinner of the compression channels and in this respect the process of testing had a selective effect. The individual results therefore reflect to a degree the effect of the tolerances. It is believed, however, that in design the effect of the tolerances may be neglected.

Channels and Angles

The sections shown on Fig. 2 are typical of those found in the most important girders of Zeppelin type airships and are representative of those used in the girders of the ZR.1. The particular sections illustrated were developed as copies of sections used in the L.49 and no attempt was made to originate new ones. The figure shows the characteristic form of the Zeppelin channel and angle, with the flange broken down by a bend so that the width of the flat surface is reduced and the resistance to buckling increased.

The results of compressive tests of the channels and angles shown in Fig. 2, made at the Massachusetts Institute of Technology at the request of the Bureau of Aeronautics, are shown in the curves of Fig. 8. With a few exceptions each spot represents a mean fibre stress from three specimens. All specimens were tested as free-ended columns fitted with end fixtures of hemispherical form. The failures of the thinner sections were indifferent; the specimens in some cases failed in three different ways for the same load. As the thickness increased the failures took on a common character and deflections were uniformly in the direction of the least radius of gyration.

The fibre stresses and values of l/r for these sections were determined from the actual sectional areas. These differed only slightly from the nominal values computed from the data of Fig. 2. The results shown in Fig. 8 will be found to be in fair agreement with those reported on pages 70 to 71 of the "Report on the Materials of Construction Used in Aircraft and Aircraft Engines" of the Aeronautical Research Committee (1920). The use of knife-edge end fittings with some of the specimens of this report probably imposed a certain degree of restraint on the sections and somewhat controlled their failure.

Formulæ for the computation of the average failing stresses to be expected of angles and channels of this type, when tested as pin-ended columns, have been proposed by Messrs. Vickers, Ltd. These formulæ have been applied to the sections of Fig. 2 with unsatisfactory results. In Table 3 will be found the predicted values for certain sections which may be compared with those recorded in the tests at the Massachusetts Institute of Technology and shown on Fig. 8, and those obtained as the average failing stresses in the three types of loadings (Table 6).

Attempts to find a method of designing channels and angles of this type so that it will be possible to predict by calculation their behaviour from the properties of the sections have so far been unsuccessful. Apparently the ratio of the width of the outstanding leg to the thickness is one of the principal factors

in determining the strength of such sections. If the results shown on Fig. 8 are plotted for each value of l/r with values of d/t as abscissæ (where d is the inside depth of the channel or angle from the line joining the edges and t is the thickness) it will be found that each type of section settles into a sort of family and that the two channel families are quite closely related, while the angle family is quite different. If the values from the lipped channels of the Aeronautical Research Committee Report are added it will be found that they depart from those of Fig. 8 for the lower values of l/r, but that as the value of l/r increases the agreement improves.

The failure of channels and angles of this character under column loads can proceed in several different ways. Primary failure may take place by the bending of the entire column either in the direction of least moment of inertia or even transversely to it. Primary failure may also occur through a twisting of the section as a whole and without any buckling of the flanges. In addition, there are the secondary failures, where one or both flanges buckle before the rest of the column shows very much distress. For each of these methods of failure there can be plotted a corresponding curve showing the relation of stress at failure to the l/r. The appropriate value of r must be taken according to the direction of failure. Plotting these curves it will be found that they intersect or interlace and that if those portions of each be taken which give the lowest values of the fibre stress for the values of l/r there will result a curve having a series of nodes, but through which a mean curve can be drawn approximating the form of the curves shown on Fig. 8. In other words, curves of the character of those on Fig. 8 can be regarded only as close approximations, and in order to determine the true curves a very extensive series of experiments with controlled failures would be required. It is probable that from such curves it would be possible to develop a very accurate method for predicting the performance of almost any channel or angle of this type. Interesting interlacing curves of the type referred to above, observed in connection with column tests of tubes, are given by Southwell in Phil. Trans., Roy. Soc., London, A 213, pp. 187-258, 1913.

At present the simplest and safest method is to use sections whose properties are known, or if none are available to keep as closely to the prototypes as possible and check the design by testing samples. The observed values of the stresses for the varying values of l/r of Fig. 8 may be compared with the ultimate stress in compression observed in the channels of the various girders when tested in the three different manners. This comparison is shown in Table 6.

When tested individually the channels fail in various ways. They bend in the direction of either principal axis, twist or cripple in one or both flanges. When used in girders the failure is almost invariably of one type. This is illustrated in Figs. 10 and 11. The flattening of the lip to enable the better attachment of the lattices appears to weaken the channel, for the failure is an exaggeration of the distortion existing in the channel before loads were applied to it.

This suggests the use of tubing instead of channels. The possible benefits from such construction have been appreciated for a long time, but the mechanical difficulties involved in assembly are serious. By the use of tubes in place of channels it is possible that the design of girders of the types used in rigid airships will be found somewhat simpler. The column strength of tubes is in general more definite and it will be easier to determine the rules governing their application. It would, therefore, appear that the most promising method of improving the girders in rigid airships is to use tubing in place of channels and angles.

Lattices

The lattices used in the Zeppelin type girders are stampings made from sheet, and the form is illustrated in Figs. 10 to 14. These lattices have been described so often that description here is unnecessary.

Tensionmeter readings taken at the Bureau of Standards on lattices in girders under load show that the stresses in the lattices are extremely small. Values of 2,000 to 3,000 lbs. per square inch are the maximum met with. The only failures encountered in the testing of 150 girders were those which occurred in the thinnest lattices and at shears in the girders far beyond the loads met with in practice. A design for a new lattice may well follow the Zeppelin prototype as to the depth of rib and bringing the edges up to the neutral axis. It should have the proper slope, according to the depth of girder, to break down the length of the longitudinal members to give the l/r ratio desired. It is believed that the high centre

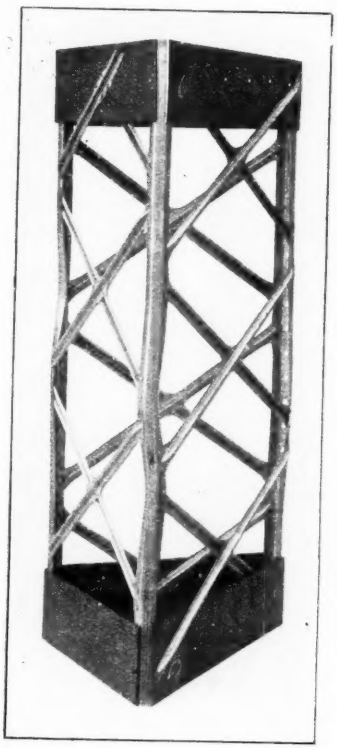


FIG. 10.
Typical deformation of channels just before failure.

rib should be carried out as far as possible. This adds to the stiffness of the channel when in place and assists it to carry compressive loads. It is found that the exact form of any lattice is considerably influenced by the necessity for providing certain radii and "draws" in the dies.

The Girders

The Bureau of Standards has just completed the testing of a series of girders taken from those being incorporated in the structure of the ZR.1. These tests were requested by the Bureau of Aeronautics for the double purpose of (1) deter-

mining the actual strength of the girders going into the ship, and (2) obtaining information for use in the design of future ships. All the information from these tests is summarised in Tables 2, 3, 4 and 5. It is believed that these are the first comprehensive tests of girders of this type to be published.

The number of girders tested was 150 and included three typical specimens of the braced main transverse. The girders may be identified by their type numbers and represent the following principal members in the ship:—

Main transverse ...	Intermediate longitudinals, top of ship
21, 23	Main longitudinals.
52, 53, 54, 55 ...	Intermediate transverses.
31B, 32	Main transverse, across bottom of keel.
42	Girders supporting fuel tank and ballast bags.
71, 72	Intermediate longitudinals, bottom of ship.
91, 92	King post braced section between main longitudinals.

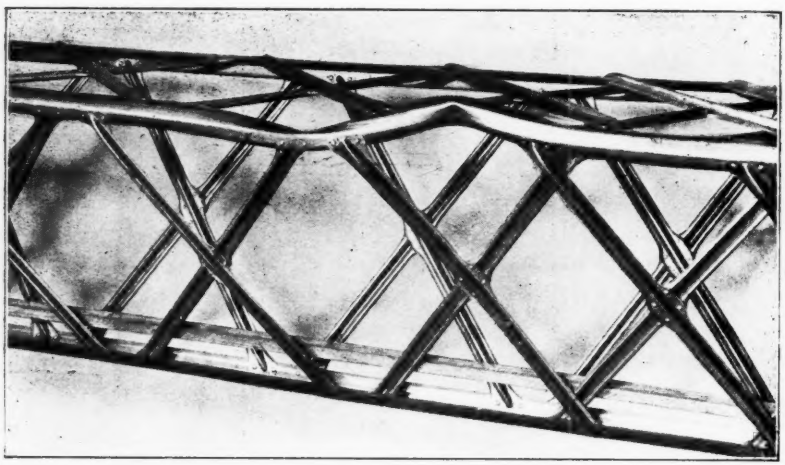


FIG. 11.

Typical failure of channels.

The important dimensions of the different types are shown on Figs. 2 to 7 inclusive; their properties are given on Table 1.

The girders were made at the Naval Aircraft Factory, Navy Yard, Philadelphia, Pa. As far as possible they were representative of the practice followed in the construction of similar girders for the ZR.1. The same methods of manufacture, assembly and riveting up were followed. Because of the extremely close tolerances used in the manufacturing processes, the girders were found to be very straight, but no more so than the girders actually used in the ship.

In order to prevent deflection of the ends during test the girders were fitted with steel end plates forming triangular boxes which were filled with cement mortar. Bolts through these cement blocks still further strengthened the ends.

For the column and combined beam and column tests, the ends were fitted with triangular bearing plates which were recessed at the theoretical neutral axis of the specimen to receive the ball of a special compression block. For the beam tests a cylindrical steel rod was inserted transversely through the end plates at

the neutral axis of the specimen. The general form of the end fixtures can be seen in Figs. 12 and 13.

The column and combined beam and column tests were made on the large horizontal Emery machine at the Bureau of Standards. The beam tests were made on the apparatus shown in Fig. 12, which was constructed for the purpose.

Most of the girders were tested as 5 metre and 10 metre lengths. The frame spacing of the ZR.1 is 5 metres, with main transverses at 10 metre spaces. The tests were intended to throw light on the behaviour of the girders under the different kinds of load to which they might be subject. Girders expected to sustain transverse loads in the ship were tested as beams, as columns, and with combined beam and column loading. In the last case the transverse load was

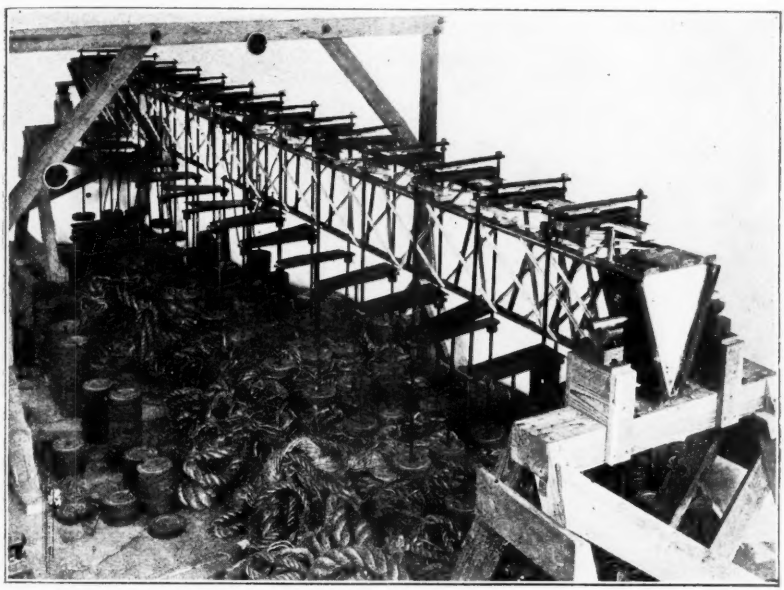


FIG. 12.

Beam test in progress.

intended to reproduce the maximum transverse load which might be expected and the test determined the maximum column load the girder would carry under this condition.

The test data are summarised in Table 2, which gives the loads and observed deflections for each type of loading. In Table 3 the beam data are analysed and the values of the fibre stress at failure and of the apparent modulus of elasticity are determined. In Table 4 the column data are analysed and values of the average end stress, P/A, and total stress at failure, taking into account the deflections, are obtained. Values of the average end stress, f_j, to be expected according to Johnson's formula and the average stress, f_e, according to Euler's formula, using the corresponding values of E of Table 3, are given for comparison with P/A. The values of P/A have been plotted on Fig. 9 and the curve of f_e drawn for a uniform value of 10,000,000 lbs./in.^{2}.

In Table 5 the results of the tests under combined beam and column loading are analysed, stresses at failure determined, and values showing the stresses to be expected by Morley's formula given for comparison.

Attention is invited to the fact that the deflections in both the direction of least radius of gyration and at right angles to it were taken for both column and combined beam and column loadings. The influence of these deflections in running the fibre stress in compression in the lengths of channels between lattice points up to values comparable with those found in the tests of the channels as ball-ended struts of the same l/r is apparent. If the sums of the end stresses at failure, P/A, and the corresponding stresses due to vertical bending, f_v, be compared with the total failing stresses computed, using Morley's formula, they will be

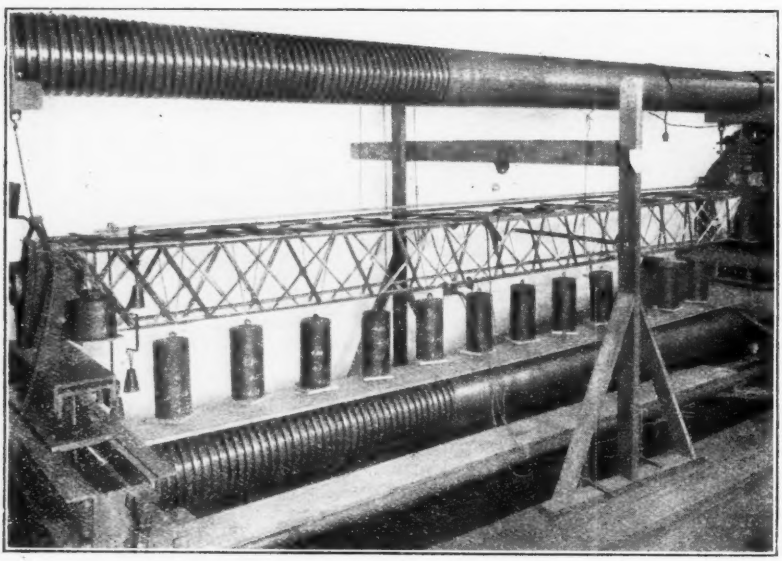


FIG. 13.

Girder under combined beam and column load.

found to be in remarkable agreement. The differences between the total stresses at failure and those by Morley's formula are due to the fact that, as there applied, the formula considers deflection in the vertical direction only.

For L/R ratios of less than 65 the failures were almost all due to secondary failure and the buckling of one of the compression channels. The exceptions were certain beam tests where failures occurred from the buckling of an end lattice. These will be referred to later. The buckling of the channels was sharply localised and, although previous to failure the sinuous distortion of the channels under compressive loads could be seen for nearly the whole length of the girder, after failure and release from load there was no distortion except in the immediate neighbourhood of the failure.

In the case of the members having L/R greater than 65, and especially in those where it was greater than 100, the failure was due to the bending of the girder as a whole. In some of the column tests the load was carried beyond the maximum load with no signs of failure; then the column suddenly collapsed under

a diminished load. Here again the channels returned to their original condition without sign of injury except in the immediate neighbourhood of the failure.

The 55D-3 was an exception to the flexural failure. This was tested as a 5-metre beam. The lattices were so closely pitched that the yield point of the material was reached before critical instability of the channels was produced. The maximum computed stress in tension f_t reached 38,200 lbs. per sq. in. and the beam deflected downward in a series of runs and stops similar to the peculiar stepped stress strain curve of duralumin. After failure permanent elongation

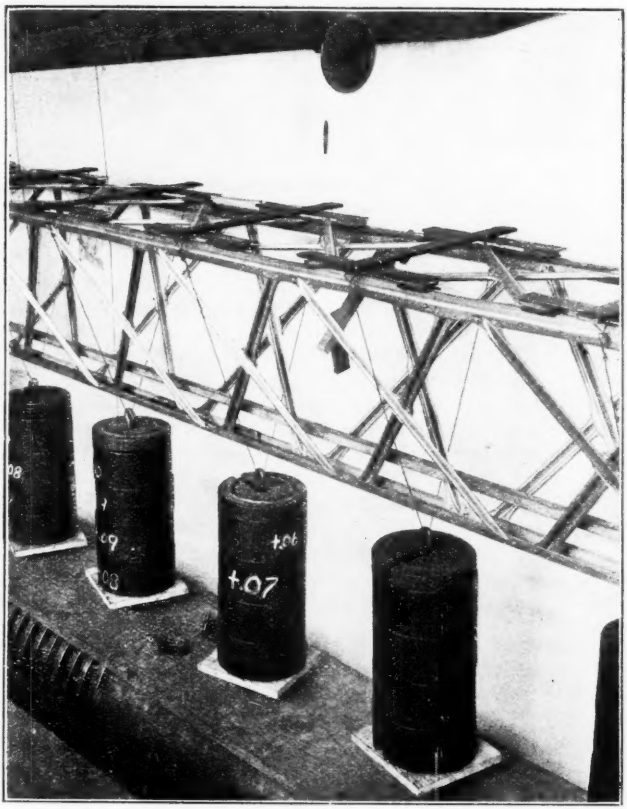


FIG. 14.

Beginning of deformation of channels in girder under combined beam and column load.

and compression were found in the apex and base channels. The characteristic bending was, however, observed in the compression channels and the final failure was similar to that of the other beams. In the case of the 52b 5-metre beam no evidence of other than flexural failure could be found, although the fibre stress of $f_t = 36{,}050$ lbs. per sq. in. seems high.

Because of the flexural character of the failure of typical girders, it is believed that their strength is dependent almost entirely on those factors of

design which determine the different types of critical instability under load which they can assume. There may be:—

(1) Flexural failure as a whole, which depends on the maximum and minimum slenderness ratios and on the type of connections and loading.

(2) Flexure and twisting of the channel as a whole between lattice points, which depends on the slenderness ratios of the channel, its torsional rigidity and the rigidity of lattice connections. This will be referred to in detail later.

(3) Buckling of the end lattices due to shear. The shape and thickness of the lattice is the governing factor here.

(4) Spreading and crumpling of the channels. This was noticed in some early experimental girders not reported here. A slight spreading was also noticed in these tests, but it is believed that it does appreciably affect the strength of the girders.

The nature of the final collapse at failure is, however, not determined by the factors mentioned above. This collapse may take various forms, crumpling and tearing of lattices, or of channels, or shearing of rivets, which depend on the yield point and ultimate strength of the various parts, but do not influence the strength of the girder under load.

With possibly two exceptions the failures were typical flexural failures due to the establishing within the elastic limit of a state of critical instability, either in the girder as a whole or in one of the parts. Consequently the ultimate loads are determined by the modulus of elasticity and the geometrical dimensions of the girders. So long as the ultimate strength and yield point of the material in tension and compression are greater than the critical stresses they have no influence on the strength. With two exceptions (5-metre beam tests 52b and 52 D-3) the computed maximum stresses were:—

Tension, $f_t = 33{,}090$ lbs. per sq. in. (10m., 55a, Table 3).
Compression, $f_c = 29{,}660$ lbs. per sq. in. (5m., 72a, Table 3).

While higher than the yield point specified for the material of which the girders was made, these are lower than the yield point actually obtained, which, as stated in an earlier portion of this discussion, is about 35,000 lbs. per sq. in.

One of the peculiar characteristics of the Zeppelin girder is the manner in which the lattice crosses are arranged around the girder. This is illustrated in Fig. 3. At first sight one would expect this "spiraling" to reduce the strength of the girder because of the unbalanced support afforded the channels by the lattices. This opinion was apparently justified by the results of earlier tests of experimental girders of somewhat different type. The spiraling was retained in the girders used in the ZR.1, but in order to determine its effect, three girders of the same size as those used in the main longitudinal were constructed with lattice crosses opposite. The tests of the two types of girders bring out the fact that the girder with the spiraled lattice crosses is stronger than that with opposite latticing. The effect of the spiraling is to reduce the l/r of the channel between lattice terminals from the full distance between the terminals, on one flange of the channel, to varying lesser values. These values are shown in Fig. 7. In this figure l_1 is the "pitch" of the lattice crosses, that is the distance from intersection to intersection, along the girder, which is the same on all three sides. Due to the spiral the distance between the lattice crosses on two adjacent sides is always 2/3 of l_1. The greatest distance between lattice ends on the same flange of the channel is that between the two ends of the lattices forming a cross, or l_2. Considering the lattices on both flanges of the channel the greatest distance in the triangular girders is normally $l_2 - 1/3$ of l_1.

Each of the above lengths, l_1 and l_2, can be regarded as an unsupported length, and by means of the curves of Fig. 8 an approximate ultimate fibre stress found for the section used with the corresponding l/r. The real degree of freedom is, of course, indeterminate in each case, but it will be noted by referring to Table 6, where the values are tabulated, that there is a remarkable agreement between the average compressive stress sustained by the channels as ball-ended columns of length l_1 and the average ultimate stress in failure found in the corresponding sections, whether the girders in which they are used are tested as beams, columns, or under combined beam and column loads. The exceptions are in the cases where the L/R of the girder is above 80. The differences grow more marked as L/R grows greater. This evidently reflects the greater tendency of the girder to fail as a whole as an Euler column.

The manner in which these girders fail reflects the "spiraling" of the lattices previously referred to. As the deflections proceed, the channel takes the sinuous form shown in its beginning in Fig. 14, well developed in Fig. 10 and after failure in Fig. 11. It will be noted in Fig. 11 that at both points of failure the twist has been inward and toward closing the channel. This is characteristic of failure in all cases.

The lattices also pick up some of the column loading. This is shown by the fact that tension meters on the channels of a column will consistently show differences of stress between the middle and ends of the column. The loads picked up by the lattices cause eccentric loads in the channels at the points of attachment and help to produce the sinuous deformation. The same is true of the longitudinal components of the loads in the lattices in beams.

A further source of this eccentric loading on the channels of columns is found in the shear deformations of the column. This was greatest in the columns with lowest l/r ratios. While the columns were being tested it could be seen that the ends of these columns were not at right angles to the tangents to the curve of bending.

In the 5-metre beam tests of the type 54a, 54b, 55b, 55C-2, 55C-3, 71c. and 92a girders the failure was by buckling of a lattice at the end of the girder. This lattice failed in compression and its deflection before failure was very great, carrying with it the tension lattice forming the other member of the cross. In all but 55b the lattices were of the thinnest gauge. Comparing the loads carried by the 55b, 71c, or 92a with those carried by the others of their type it will be seen that at failure they were sustaining practically the same load. Evidently in these types we have reached a balance between the strength against local buckling of the compression channels and the strength against buckling of the end lattices due to shear.

The loads sustained by the 5-metre type 54 girders, as beams and as combined beams and columns, were as follows:—

		Combined load.	
	Beam load.	Beam.	Column.
54a	1909	952	2460
54b	1603	952	2800
54c	2088	952	2830

As a beam, the 54a and 54b failed from the collapse of a lattice at the ends, while the 54c showed the usual channel failure. However, the figure of 1,909 lbs. for the 54a beam is so near the 2,088 for the 54c that it is evidence the beam is only slightly weaker in shear than in the compression strength of the channels. In the first two cases this type sustains, as a simple beam, twice as much as the constant transverse load fixed for it as a combined beam and column. Under this latter loading the failures were all in the channels as might be expected.

The transverse portion of the load in the combined beam and column loading

was intended to represent the maximum which might be expected on the girder in the ship. This loading was also on a beam of a single span, while in a ship the girder forms one span of a continuous beam of many spans. To reproduce this condition we should fix the ends of the specimen in direction and the bending moment at the middle of the span instead of being 1/8 WL becomes 1/24 WL and at the ends 1/12 WL. The end shears remain the same, 1/2 W each.

The moment at the end of the span would cause compression in the vertex channel and tension in the base. This condition was not reproduced in the tests at the Bureau of Standards and no information as to the behaviour of a girder under it can be given.

If those of the type 52, 53, 54 and 55 girders, which carried the greatest loads as simple beams, be assumed to be transformed into panels of continuous beams, we can make a comparison between the moments and stresses at the mid-length in the simple beam and at the support in the continuous beam as follows:—

COMPARISON BENDING MOMENTS AND STRESSES.

Girder.	Simple beam. At mid-length.		Continuous beam. At centre of support.			At 9in. from centre.	
	M	Max. f_c	M_s	I/y	f_{sc}	M_{sa}	f_{sac}
52b	44,340	22,130	29,600	1.231	24,000	22,170	18,000
53b	43,530	21,730	29,000	1.496	19,400	21,760	14,550
54c	53,400	21,710	35,600	1.496	23,800	26,700	17,850
55c	53,200	21,630	35,500	1.714	20,700	26,600	15,500

I and y from Table 1. Values of f_c, f_{sc} and f_{sac} in lbs./in.^{2}.
M and f_c (at failure) from Table 3. Values of M, M_s and M_{sa} in inch lbs.
$M_s = 66.7\%$ of M. $M_{sa} = 50\%$ of M.

The moments and stresses at mid-length of the continuous beams are one-third the corresponding values for the simple beams and very small. Over the supports, however, we get increased stresses. This support is not a point but a substantial joint where the transverse meets the longitudinal. The channels are spliced there and a large amount of extra material added in the way of doubled or closer spaced lattices. We are, therefore, justified in computing the maximum moment in the girder proper at some distance from the theoretical support. This is a common practice in plate girder work. From a study of the bending moment curve we find that at a distance of 9in. from the centre of support of these girders in the ZR.1 the bending moment falls to 50 per cent. of the simple beam bending moment. The resulting bending moments and stresses are included above.

From an inspection of the additional structure involved in a joint, and considering the increased strength of the latticing, we may safely conclude that the vertex channel at a joint is exceptionally well stayed. It is impossible for it to deflect in the joint structure, and just outside the extra lattices, closely spaced, afford almost equal support. Hence it is believed that any failure of the vertex channel in or very near a joint will be a strictly compression failure due to the elastic limit having been exceeded.

No mention has been made of the support against lateral deflection—undoubtedly considerable—afforded to the longitudinals by the netting wires and shear wire intersections. This may be expected to make the strength of girders in the ship even greater than the tests indicate.

Because of the peculiar character of the channel failure produced by the "spiralled" lattices, it must not be thought that re-locating the lattices to avoid this type of failure will necessarily increase the strength of the column. This

is shown by the results from the column tests of 55 and 55B. The component parts of both girders are the same, but in the latter the lattices are opposite instead of spiralled.

When tested as columns the ultimate loads in lbs. carried were as follows:—

55	a	...	5,600	55	B-1	...	4,780
	b	...	5,160	55	B-2	...	5,110
	c	...	5,900	55	B-3	...	4,640
Mean		...	5,550	Mean		...	4,810

This loss of strength can be explained by the lack of the thing that caused the sinuous deflections in the standard girder, the restraint imposed by the lattices. While the spiralled lattices were forcing the channel to deflect sidewise and twist they were preventing it from deflecting in the direction of its least radius of gyration and from failing at the lowest load. Also they were breaking it up into a series of short lengths by the forced deflections. Removing these restraints by bringing all the lattices together at one point on the channel, while it balances the lateral forces on the channel, leaves it free to deflect in the direction of the least radius of gyration and as a strut of the full "pitch" length. Any circumstance which might alter this would increase its strength. Possibly the high value of 5,110 lbs. for 55 B-2 was due to a lateral deflection of the channels established in part.

The mean values of the apparent modulus of elasticity, E, for each type of girder have been plotted against values of L/R on Fig. 9. The positions of the plotted points bring out very clearly the influences of length, variation in sectional area and shear resistance due to strength of lattices. Beginning with the lowest value of E, which is that for type 52, it will be seen that with each increase in sectional area, or of lattice thickness, or of L/R, there comes an increase in the value of E. Equally effective is the addition of extra latticing.

The comparison of 55, 55C and 55D is interesting. All three have the same channels. 55 has lattices .023in. thick on the sides and .019in. thick on the base, and set with the usual "spiral." 55C has lattices .019in. thick on the sides and base, but "opposite," that is with no "spiral." 55D has the .019in. lattices with no spiral but doubled, that is at half the pitch of 55C. The value of E from 55 is greater than that from 55C, but less than that from 55D. The shear restraint of the opposite lattices of 55C was less than that of the spiralled lattices of 55, but in the latter in turn the shear restraint of the spiralled lattices was less than that of the doubled opposite ones.

The main transverses tested were intended to reproduce the portion of a main transverse between two main longitudinals. The general form is shown on Fig. 25. The main girder was a type 45, which is the same in dimension as the 42A, but with B-6 vertex channel and B-5 base channels. The lattices were .027in. thick on all sides. The brace girders were type 12 equilateral girders with B-1 channels at each vertex and lattices .019in. thick. The length of the side of the transverse section was 6.26in. and the lattices were 9.843in. centre to centre and 5.83in. between ends of the same cross. The section was tested only as a ball-ended column. A short dummy intermediate longitudinal girder was fitted and the test apparatus arranged to reproduce the restraints actually present in the ship. Deflection was free radially only.

The observed loads and deflections were as follows:—

Girder.		At Pro. Lim.		At Ultimate.	
		Load.	Deflection	Load.	Deflection.
Main transverse	a ...	8,000	0.618	10,120	0.872
,, ,,	b ...	7,000	0.580	8,720	0.773
,, ,,	c ...	7,500	0.616	9,000	0.831
Mean	...	7,500		9,280	

Load in lbs.
Deflection in inches (Radial).

Application to Design

The stresses to be expected in the girders under column loads were computed by the formula proposed by Messrs. Vickers, Ltd. The formula and the values it gives for certain girders will be found in Table 4. Their agreement with observed values is good in some cases, but not so in others. It was concluded that it was not strictly applicable.

It is believed that the use of the data given in this section will make it possible to estimate the strength of existing new girders, or new designs, with considerable success. The weak links are the data for the design of channels or angles of some untested section, and the lattices. The data of Fig. 8 may, of course, be supplemented by any available data regarding the strength of similar sections, or, if the sections are available, they can be tested in a short time. It is even possible to bend up experimental channels of new types by hand and test them.

Sufficient data regarding the lattices has not yet been obtained. The form and general properties are well known, however, and it should be possible to get on quite well by falling back on published information and ordinary design practice.

For the design of a beam to carry purely bending loads the process would be the familiar one of selecting by judgment the proportions of the beam section and determining approximate values of its properties. The required value of I/y would be determined from the bending moment and the assumption of a fibre stress in compression from data, such as from Fig. 8, as to the behaviour of the available channels when tested as free-ended columns. The process then becomes one of harmonising the value of I/y with the properties of the beam section, and arranging the lattices to provide the l/r necessary to develop the assumed fibre stress. The end shears would have to be taken care of by falling back on experience and judgment. Doubling the lattices—halving the pitch—at the ends and in the case of beams of very great stiffness increasing the lattice thickness also would, of course, be considered.

In the case of a column a tentative design would be prepared, following existing columns if possible, and a lattice pitch assumed. Data as to the apparent value of E to be expected, such as Fig. 9, and as to the compressive strength of the channels to be expected, such as Fig. 8, would be referred to and values selected. If L/R was over 100, Euler's formula would be used as an approximation of the value of P/A to be expected; if L/R was under 100, Johnson's formula would be applied. The design figures would be changed as necessary to adjust the differences.

The main and intermediate longitudinals must carry transverse loads, and on occasion either compressive or tensile loads, according to the condition of the ship, and the forces acting on its structure as a whole. The analysis of the effect of the transverse and tensile loads can be easily effected by the usual methods. For the analysis of combinations of transverse and compressive loads, it is suggested that the generalised theorem of three moments be used. This may be done conveniently by the use of the functions developed and tabulated by Mr. Berry. An approximate design can be made by estimating the position of the points of inflection of the girder without end load, assuming they do not change position when an end load is applied, and assuming the continuous beam to be broken up into a simple beam and two cantilevers. From the approximate design a tentative design can be prepared. If the points of inflection do not remain fixed, as assumed, they will move toward the supports which will help the girder over the support. In the middle of the span the girder will probably be overstrong to begin with and no harm will be done. The tentative design may then be analysed by the generalised theorem of three moments and the Berry

functions. The fibre stresses in the channel elements at the critical points—points of support and mid-length—should then be calculated and compared with the results from tests of the channels as pin-ended columns, as in the case of an ordinary beam, to determine that the crippling load of the channel has not been exceeded.

In these computations the value of E should, if possible, be selected from the results of tests on girders of similar type. For the preliminary design of girders similar to those discussed in this section, E may be approximated either by using that found in the most nearly similar girder or by interpolation along a curve of E, which may be drawn from the values given on Fig. 9.

The shears at the supports should be determined and such provision made for them as experience and judgment indicate as proper.

Due advantage may be taken of the rapid drop from the sharp peak of moment over the support and the peak itself assumed to be absorbed in the additional material required to make the joint at the point of support. The maximum moment may, therefore, be taken as occurring at a point which is out from the theoretical point of support by an amount which the details of the joint will enable one to judge.

The tests on girders here described will be used in subsequent sections of this paper in discussing the general strength of an airship such as ZR.1.

For design purposes, it is to be noted that these tests lead to the following conclusions, which are general in so far as they apply to girders of these types :—

(*a*) The crippling by compression of angle or channel elements of a girder determines the failure of girders as beams, short columns, or under combined loading.

(*b*) Tests of angle or channel elements having slenderness ratios as determined by the girder latticing may be used to predict the failure of such elements as under (*a*).

(*c*) Long columns fail as predicted by Euler's formula, using E as obtained experimentally from deflection of the girder as a simple beam.

(*d*) Short columns fail as predicted by Johnson's formula, using E as in (*c*) and f_c as in (*b*).

References to Table 1.

w' = weight, in pounds per inch of length, of normal girder.

a_c = 2 × area of base channel.

a_t = area of apex channel for triangular girders = 2 × area of upper channel for rectangular girder.

$A = a_c + a_t$.

$y_c = a_t h / A$; $y_t = a_c h / A$.

$c_c = y_c + m_c$; $c_t = y_t + m_t$; $y_b = b/2$; $c_b = y_b + m_b$.

$I_{max} = a_c y_c^2 + a_t y_t^2 = a_c y_c h = a_t y_t h$.

$I_{min} = 1/4\ a_c b^2$ for triangular girders.

$I_{min} = 1/4 A b^2$ for rectangular girders.

These simplified formulæ suffice because the moment of inertia of a channel section about the axis through the centre of inertia is negligible in comparison with the moment of inertia of the girder.

TABLE 1.

DIMENSIONS AND PROPERTIES OF GIRDERS TESTED AT THE BUREAU OF STANDARDS (see Fig. 7).

Girder type.	Apex type.	Channel area. in.²	Base type.	Channel area. in.²	h in.	b in.	m_c in.	m_t in.	m_b in.	Weight per 5m. length w lb.	Weight per in. length w' lb./in.
21	A-1	0.0911	B-1	0.0609	7.40	6.10	0.413	0.230	0.4096	7.327	0.0372
23	A-1	0.0911	B-1	0.0609	7.40	6.10	0.413	0.230	0.4096	9.574	0.0487
42	B-3	0.0911	B-3	0.0911	8.74	10.24	0.4146	0.2106	0.4123	9.51	0.0483
52	A-1	0.0911	B-2	0.0742	13.50	10.63	0.4138	0.230	0.4104	9.222	0.0468
53	A-2	0.1108	B-2	0.0742	13.50	10.63	0.4138	0.231	0.4104	9.608	0.0488
54	A-2	0.1108	B-3	0.0911	13.50	10.63	0.542	0.231	0.4114	10.270	0.0522
55	A-3	0.1259	B-3	0.0911	13.50	10.63	0.543	0.232	0.4114	11.196	0.0569
71	N-3	0.0849	N-3	0.0849	7.50	5.51	0.246	0.246	0.246	10.763	0.0548
72	N-3	0.0849	N-3	0.0849	7.50	5.51	0.246	0.246	0.246	11.053	0.0562
91	A-1	0.0911	B-1	0.0609	9.45	7.87	0.413	0.230	0.4096	8.059	0.0409
92	A-2	0.1108	B-2	0.0742	9.45	7.87	0.4138	0.231	0.4107	8.968	0.0455
31B	B-2	0.0742	B-2	0.0742	6.81	7.87	0.4138	0.2088	0.4114	7.714	0.0392
32	B-3	0.0911	B-2	0.0742	6.81	7.87	0.4138	0.2106	0.4114	8.613	0.0437

Girder type.	a_c in.	a_t in.²	A in.²	y_c in.	y_t in.	c_c in.	c_t in.	I_{max} in.⁴	y_b in.	c_b in.	I_{min} in.⁴
21	0.1218	0.0911	0.2129	3.167	4.234	3.580	4.464	2.854	3.050	3.460	1.133
23	0.1218	0.0911	0.2129	3.167	4.234	3.580	4.464	2.854	3.050	3.460	1.133
42	0.1822	0.0911	0.2733	2.913	5.827	3.328	6.037	4.039	5.120	5.532	4.776
52	0.1484	0.0911	0.2395	5.135	8.365	5.549	8.595	10.29	5.315	5.725	4.192
53	0.1484	0.1108	0.2592	5.771	7.729	6.185	7.960	11.56	5.315	5.725	4.192
54	0.1822	0.1108	0.2930	5.105	8.395	5.520	8.626	12.56	5.315	5.726	5.147
55	0.1822	0.1259	0.3081	5.517	7.983	5.932	8.215	13.57	5.315	5.726	5.147
71	0.1698	0.1698	0.3396	3.780	3.780	4.026	4.026	4.852	2.755	3.001	2.578
72	0.1698	0.1698	0.3396	3.780	3.780	4.026	4.026	4.852	2.755	3.001	2.578
91	0.1218	0.0911	0.2129	4.044	5.406	4.457	5.636	4.654	3.935	4.345	1.886
92	0.1484	0.1108	0.2592	4.040	5.410	4.454	5.641	5.665	3.935	4.346	2.298
31B	0.1484	0.0742	0.2226	2.270	4.540	2.684	4.749	2.204	3.935	4.346	2.298
32	0.1484	0.0911	0.2395	2.590	4.220	3.004	4.430	2.618	3.935	4.346	2.298

TABLE 2.

TEST DATA SUMMARY, BEAMS.

Girder.		Length L meter.	Proportion limit. Load lb.	Proportion limit. Vertical deflection in.	Maximum load. Load lb.	Maximum load. Vertical deflection.
21	a	5	560	2.35	680	2.933
	b		530	2.23	670	2.883
	c		600	2.43	740	3.118
23	a	5	475	1.96	697	3.218
	b		600	2.36	775	3.652
	c		500	2.05	725	2.988
23	a	10	261	7.98	351	11.576
	b		296	9.15	359	11.769
	c		296	9.27	338	11.067
52	a	5	1176	1.61	1588	2.936
	b		1176	1.56	1733	3.576
	c		1176	1.70	1588	2.661
53	a	5	1323	1.51	1605	2.002
	b		1436	1.57	1701	2.063
	c		1436	1.90	1686	2.696
54	a	5	1323	1.43	1909*	2.949
	b		1258	1.41	1603*	1.896
	c		1258	1.42	2088	3.576
55	a	5	1659	1.59	2079	2.243
	b		1538	1.44	2088*	1.957
	c		1538	1.51	1958	2.172
55	C-2	5	1005	1.01	1750*	1.952
	3		1145	1.16	1745*	1.850
55	D-3	5	1285	1.14	2540	3.969
55	a	10	893	5.91	1083	7.692
	b		767	5.04	972	6.852
	c		767	4.95	1017	7.031
71	a	5	865	2.03	1285	3.004
	b		865	1.94	1400	3.584
	c		1005	2.17	1380*	3.305
72	a	5	1005	2.30	1395	3.448
	b		1075	2.43	1350	3.150
	c		1075	2.45	1485	3.415
72	a	10	455	7.53	645	11.206
	b		525	9.07	700	12.540
	c		525	8.81	665	11.860
91	a	5	865	2.22	1060	2.867
	b		725	1.90	970	2.543
	c		725	1.72	1000	2.656
92	a	5	865	1.85	1280	3.111
	b		865	1.88	1280	2.947
	c		865	1.81	1270	2.705
92	a	10	455	6.51	650*	9.860
	b		455	6.57	620	9.480
	c		455	6.60	595	7.850

* Failed near end by buckling of lattices, due to "shear" stresses. See discussion.

NOTE.—The load was applied at 14 points in 21 (5m), 23 (5m), and 23 (10m). In all others at 28 points.

TABLE 2.

TEST DATA SUMMARY, COLUMNS.

Girder.	Length L meter.	Proportion limit. Load lb.	Proportion limit. Horizontal lateral deflection in.	Max. load lb.	Failure. Load lb.	Failure. Lateral deflection in. Horizontal.	Failure. Lateral deflection in. Vertical.
21 a	5.0	2030	.080	2430	2400	.878	.368
b	,,	2400	.273	2700	2700	.774	.271
c	,,	1700	.249	2250	2210	1.496	.340
23 a	5.0	2300	.362	2490	2450	1.557	.254
b	,,	2200	.072	2600	2540	.920	.463
c	,,	1800	.224	2410	2410	1.038+	.284+
23 a	10.0	650	.189	770	740	9.410	.595
b	,,	550	.847	710	700	1.970	6.864
c	,,	550	.337	720	720	6.847	.695
52 a	5.0	4000	.067	4460	4460	.145+	.093+
b	,,	4000	.102	4520	4520	.089+	.233+
c	,,	4000	.035	4300	4300	.069+	.034+
53 a	5.0	3600	.076	4540	4540	.342+	.280+
b	,,	—	—	—	—	—	—
c	,,	4400	.203	4530	4530	.362	.194
54 a	5.0	4500	.430	4500	4500	.430	.593
b	,,	4900	.154	5045	5045	.173	.461
c	,,	4800	.076	5240	5240	.189+	.029+
55 a	5.0	5450	.238	5600	5600	.588	.282
b	,,	4800	.063	5160	5160	.168	.147
c	,,	5400	.005	5900	5900	.003+	.012+
55 A-1	5.0	4500	.070	5140	5140	.058+	.155+
2	,,	4600	.185	5330	5330	.537+	.246+
3	,,	4600	.440	5130	5130	.561+	.226+
55 B-1	5.0	4500	.275	4780	4780	.100	.021
2	,,	4900	.432	5110	5110	.005+	.003+
3	,,	4200	.135	4640	4640	.157	.327
55 C-1	5.0	5100	.106	5430	5430	.157+	.042+
55 D-1	5.0	5000	.073	7180	7180	.408+	.095+
2	,,	4500	.056	6760	6760	.365+	.103+

NOTE.—A plus sign after a deflection indicates that the deflection was observed at a preceding lower load. At the maximum load the deflection was so great that the dials ceased to operate or was increasing so rapidly that no reading could be taken.

TABLE 2.

TEST DATA SUMMARY, NOTES.

The maximum end load carried and the end load at failure are both given in the data on the tests as columns and as combined beams and columns. Both may be referred to as P in the analyses, but the headings plainly say "Maximum" or at "Failure." By noting the heading there should be no confusion. In some cases the maximum end load and end load at failure were the same. The figures in the two columns will show this.

TABLE 2.

TEST DATA SUMMARY, COLUMNS.

Girder.		Length L meter.	Proportional limit. Load lb.	Proportional limit. Horizontal lateral deflection in.	Maximum load.	Failure. Load lb.	Failure. Lateral deflection in. Horizontal.	Failure. Lateral deflection in. Vertical.
55	a	10.0	2400	.362	3080	3080	3.715	.207
	b	,,	2400	.386	3060	2790	3.841	.325
	c	,,	3000	.132	3270	3100	1.930	.028
71	a	5.0	4000	.237	5190	4930	2.053	.501
	b	,,	5000	.059	5720	5360	1.670	.299
	c	,,	5000	.127	5640	5300	1.513	.129
72	a	5.0	5100	.254	5500	5440	1.259	.345
	b	,,	5500	.167	5880	5600	1.379	.497
	c	,,	5000	.260	5540	5300	1.537	.243
72	a	10.0	1500	.557	1720	1600	11.912	.480
	b	,,	1200	.945	1650	1650	9.665	.342
	c	,,	1200	.210	1720	1720	10.965	.620
91	a	5.0	3700	.009	4120	4120	.141+	.128+
	b	,,	3000	.180	3400	3400	.420	.145
	c	,,	3300	.101	3780	3780	.333+	.084+
92	a	5.0	4400	.321	4500	4500	.636	.176
	b	,,	4500	.080	4690	4690	.237+	.255+
	c	,,	4200	.023	4420	4420	.350	.251
92	a	10.0	1100	.501	1460	1430	6.804	.335
	b	,,	1100	.292	1470	1460	5.037	.373
	c	,,	1200	.462	1460	1430	5.180	.625
42	A-a	5.0	5000	—	6010	6010	.273+	.117+
	b	,,	4500	.166	5130	5130	.327+	.327+
	c	,,	6200	.086	6500	6500	.186	.092
31	B-a	3.0	—	—	5340	5340	.067+	.266+
	b	,,	—	—	5090	5090	.055+	.089+
	c	,,	4300	.031	4900	4900	.110	.207
32	a	3.0	—	—	5590	5590	.059+	.181+
	b	,,	—	—	5620	5620	.002+	.179+
	c	,,	4500	.076	5160	5160	.161+	.255+

NOTE.—A plus sign after a deflection indicates that the deflection was observed at a preceding lower load. At the maximum load the deflection was so great that the dials ceased to operate or was increasing so rapidly that no reading could be taken.

TABLE 2.

TEST DATA SUMMARY, COMBINED COLUMN AND BEAM LOADINGS.

Girder.	Length L meter.	Transverse load (vertical) lb.	Max. end load lb.	End load lb.	Failure. Deflection in. Horizontal.	Vertical.
21 a	5	380	1180	1180	.066+	1.822+
b			1020	1020	.493	1.795
c			1000	1000	.550	1.893
23 a	5	380	1150	1150	.398+	1.710+
b			1365	1365	.525+	1.785+
c			1210	1210	.696+	1.867+
23 a	10	190	210	210	.240+	5.010+
b			150	150	1.870	5.770
c			180	150	2.420	6.670
52 a	5	710	2560	2560	.162+	1.360+
b			2380	2380	.811+	1.539+
c			2400	2400	.849+	1.590+
53 a	5	710	2450	2450	.256	1.155
b			2400	2400	.343	1.166
c			2550	2550	.551	1.219
54 a	5	952	2530	2460	.485	1.640
b			2840	2800*	.992	1.678
c			2830	2830	.665+	1.610+
55 a	5	952	3500	3500	.136+	1.011+
b			3450	3450	.133	1.000
c			3370	3370	.620+	1.449+
55 a	10	475	1850	1850	.033+	4.100
b			1450	1450	1.314	3.915
c			1390	1390	1.317	3.830
71 a	5	570	3280	3230	.115	2.182
b			3300	3300	.322	1.995
c			3140	3140	.449	1.968
72 a	5	570	3280	3280	.480+	1.874+
b			3100	3100	.307	1.839
c			2180**	1960	.495	1.770
72 a	10	286	1030	1030	3.704	7.815
b			930	930	1.807+	7.085+
c			960	960	5.605	8.590
91 a	5	380	2050	2050	.060+	1.340+
b			2230	2230	.279+	1.302+
c			1990	1990	.038+	1.347+
92 a	5	380	3020	3020	.358+	1.156+
b			3290	3290	.065+	1.299+
c			3110	3110	.392+	1.261+
92 a	10	190	940	940	4.220	4.565
b			860	860	2.395+	4.550+
c			1010	960	2.863	5.114

* By an evident mistake this is given as 2600 on log sheets.

** Damaged longitudinal channel repaired. See discussion.

NOTE.—A plus sign after a deflection indicates that the deflection was observed at a preceding lower load. At the maximum load the deflection was so great that the dials ceased to operate or was increasing so rapidly that no reading could be taken.

TABLE 3.

ANALYSIS OF BEAM DATA.

Girder.		Length. m.	Bending Moment (M). At pro. limit in lb.	Bending Moment (M). At ult. load in lb.	Maximum Stresses. Compressive. At pro. limit f_c lb./in.².	Compressive. At ult. load f_c lb./in.².	Tensile. At pro. limit f_t lb./in.².	Tensile. At ult. load f_t lb./in.².
21	a	5.0	14880	18030	16510	20000	22070	26740
	b	,,	14090	17770	15630	19710	20900	26350
	c	,,	15930	19090	17670	21190	23630	28320
23	a	5.0	12700	18530	14090	20560	18840	27490
	b	,,	15980	20580	17740	22830	23710	30520
	c	,,	13360	19260	14830	21370	19820	28580
23	a	10.0	14640	19370	16250	21490	21720	28730
	b	,,	16480	19790	18280	21950	24450	29350
	c	,,	16480	18680	18280	20730	24450	27720
52	a	5.0	30160	40610	15060	20270	24530	33020
	b	,,	30160	44340	15060	22130	24530	36050
	c	,,	30160	40650	15060	20290	24530	33040
53	a	5.0	33910	41090	16930	20510	22670	27470
	b	,,	36790	43530	18370	21730	24600	29090
	c	,,	36790	43150	18360	21540	24600	28850
54	a	5.0	33930	48850	13790	19860*	22680	32660*
	b	,,	32270	41060	13120	16690*	21520	27450*
	c	,,	32270	53400	13120	21710	21570	35700
55	a	5.0	42500	53200	17280	21630	25000	31300
	b	,,	39420	53420	16030	21720*	23200	31430*
	c	,,	39420	50120	16030	20370	23200	29480
55	C_2	5.0	25860	44820	10510	18260*	15210	26370*
	C_3	,,	29420	44690	11960	18170*	17310	26300*
	D_3	,,	32980	64930	13390	26400	19370	38200
55	a	10.0	46560	56240	18930	22860	27400	33090
	b	,,	40150	50590	16230	20570	23620	29760
	c	,,	40150	52880	16320	21500	23620	31110
71	a	5.0	22280	32970	17360	25690	17360	25690
	b	,,	22280	35900	17360	27970	17360	27970
	c	,,	25850	35390	20140	27570*	20140	27570*
72	a	5.0	25850	35780	20140	27870	20140	27870
	b	,,	27640	34640	21530	26980	21530	26980
	c	,,	27640	38070	21530	29660	21530	29660
72	a	10.0	24250	33920	18890	26430	18890	30430
	b	,,	27820	36720	21670	28610	21670	28610
	c	,,	27820	34940	21670	27220	21670	27220
91	a	5.0	22220	27180	19300	23620	25750	31570
	b	,,	18650	24890	16110	21620	21660	28910
	c	,,	18650	25650	16110	22290	21660	29800
92	a	5.0	22240	32800	15860	23390	21240	31330
	b	,,	22240	32800	15860	23390	21240	31330
	c	,,	22240	32550	15860	23210*	21240	31090*
92	a	10.0	24050	33970	17150	24230	22970	32450
	b	,,	24050	32450	17150	23140	22970	30990
	c	,,	24050	31170	17150	22230	22970	29770

TABLE 3.

ANALYSIS OF BEAM DATA.

Girder.	Length.	Vickers' formula f_v lb./in.². Primary failure.	Secondary failure.	Apparent modulus of elasticity (E) in bending lb./in.².	Apparent modulus of elasticity (E) lbs.in.² Means
21 a	5.0	—	—	8,960,000	
b	,,	—	—	8,940,000	9,053,000
c	,,	—	—	9,260,000	
23 a	5.0	—	—	9,100,000	
b	,,	—	—	9,580,000	9,318,000
c	,,	—	—	9,210,000	
23 a	10.0	—	—	10,380,000	
b	,,	—	—	10,190,000	10,206,000
c	,,	—	—	10,050,000	
52 a	5.0	—	—	7,360,000	
b	,,	—	—	7,590,000	7,303,000
c	,,	—	—	6,960,000	
53 a	5.0	—	—	7,850,000	
b	,,	—	—	8,180,000	7,607,000
c	,,	—	—	6,870,000	
54 a	5.0	—	—	7,620,000	
b	,,	—	—	7,360,000	7,426,000
c	,,	—	—	7,300,000	
55 a	5.0	29020(6)	26190(6)	7,950,000	
b	,,	53960(7)	33030(7)	8,140,000	7,903,000
c	,,	62660(8)	35200(8)	7,620,000	
55 C_1	5.0	—	—	7,590,000	7,552,000
C_2	,,	—	—	7,520,000	
D_3	,,	—	—	8,590,000	8,590,000
55 a	10.0	—	—	9,390,000	
b	,,	—	—	9,520,000	9,517,000
c	,,	—	—	9,640,000	
71 a	5.0	—	—	9,130,000	
b	,,	—	—	9,560,000	9,529,000
c	,,	—	—	9,900,000	
72 a	5.0	—	—	9,330,000	
b	,,	—	—	9,450,000	9,393,000
c	,,	—	—	9,400,000	
72 a	10.0	—	—	10,730,000	
b	,,	—	—	10,200,000	10,478,000
c	,,	—	—	10,510,000	
91 a	5.0	33070(6)	25180(6)	8,690,000	
b	,,	41170(7)	28960(7)	8,510,000	8,858,000
c	,,	41910(8)	29430(8)	9,380,000	
92 a	5.0	46570(6)	29950(6)	8,570,000	
b	,,	62270(7)	35150(7)	8,420,000	8,571,000
c	,,	63800(8)	35810(8)	8,730,000	
92 a	10.0	—	—	10,530,000	
b	,,	—	—	10,430,000	10,448,000
c	,,	—	—	10,390,000	

TABLE 3.

(1) The computed stresses marked * should not be used in estimating strength of longitudinals to resist compression. See Table 2 and discussion.

(2) $M = \{ (n+2)/(n+1) \} (WL/8) (wL/8) = 0.12931\ LW + 0.125\ Lw$

where W is total load and w weight of the beam (beams 21 and 23 were loaded at 14 points and for them M is $0.13333\ LW + 0.125\ Lw$).

The correction factor $(n+2)/(n+1)$ is due to the fact that the load was distributed in n (an even number) equally spaced equal concentrated loads instead of the desired uniformly distributed load.

(3) $f_c = My_c/I_{max}$; $f_t = My_t/I_{max}$

Neither of course gives the correct maximum stress since the channels neither remain straight between lattices nor bend as a portion of a solid beam. Comparison of these results with design data should of course be made on a uniform basis.

(4) $E = 5\ ML^2/48\ Id$, where d is deflection in inches at proportional limit and I is I_{max}. About the axis of I_{min} the apparent Modulus of Elasticity would probably be larger and would lie between the apparent E about the axis of I_{max} and E as determined in tensile tests of channels.

(5) Vickers' formulæ for average failing stress in channel when tested as pin-ended column :—

Primary failure $1/f_v = (b/t)^2/4500 + (l/r)^2/50000$.
Secondary failure $1/f_v = (b/t)/234 + (l/r)/1200$.

Where f_v = pressure on end of section at failure in tons (of 2240 lbs.) per sq. in.
b = over all width of flange of specimen in inches (in the case of channel sections, of the side flanges of the section).
t = the flange thickness in inches.
l = the equivalent pin-jointed length of the section as a strut.
r = the least radius of gyration.

f_v, as given in Table 3, was calculated for the three different values of l.

(6) For l = distance between the lattices on the base.

(7) For l = maximum distance between any two lattices on the base channels including side lattices.

(8) For $l = \frac{1}{2}$ distance between the lattices on the base.

The last represents the true equivalent pin-jointed length. The three values (6), (7) and (8) are marked in the table with the corresponding numbers.

TABLE 4.

ANALYSIS OF COLUMN DATA.

Girder.	Length L m.	L/R.	Bending Moments at Failure (1). Horizontal. $M_h = Pd_h$ in lb.	Vertical. $M_v = Pd_v$ in. lb.	Bending Stress at Failure (2). Horizontal. f_{bh} lb./in.².	Vertical. f_{bv} lb./in.².	Stresses at Failure (3). Average end P/A lb./in.².	Total. $P/A + f_{bh} + f_{bv}$ lb./in.².
21 a	5	85	2110	880	5670	980	11270	17930
b	5	85	2090	730	5630	810	12680	19120
c	5	85	3310	750	8900	830	10380	20110
23 a	5	85	3810	620	10270	690	11510	22470
b	5	85	2340	1180	6290	1310	11930	19530
c	5	85	2500+	680+	6730+	760+	11320	18810+
23 a	10	170	6960	440+	18740	490+	3480	22710+
b	10	170	1380+	4800	3710+	5330	3290	12330
c	10	170	4930	500+	13270	550+	3380	17200+
52 a	5	47	650+	410+	820+	210+	18620	19650+
b	5	47	400+	1050+	510+	530+	18870	19910+
c	5	47	300+	150+	380+	70+	17950	18400+
53 a	5	49	1550+	1270+	1970+	640+	17520	20120+
b	—	—	—	—	—	—	—	—
c	5	49	1640	880	2080	440	17480	20000
54 a	5	47	1940	2670	2000	1090	15360	18440
b	5	47	870	2330	900	950	17220	19060
c	5	47	960+	150+	1000+	60+	17880	18040+
55 a	5	48	3290	1580	3400	640	18180	22220
b	5	48	870	760	900	310	16750	17950
c	5	48	20+	70+	20+	30+	19150	19200+
55 A-1	5	48	300+	800+	310+	320+	16680	17320+
2	5	48	2860+	1310+	2960+	530+	17300	20790+
3	5	48	2880+	1160+	2970+	470+	16650	20090+
55 B-1	5	48	480	100	490	40	15510	16050
2	5	48	30+	20+	30+	10+	16590	16620+
3	5	48	730	1520	750	620	15060	16530
55 C-1	5	48	850+	230+	880+	90+	17620	18600+
55 D-1	5	48	2930+	680+	3030+	280+	23300	26610
2	5	48	2470+	700+	2550+	280+	21940	24770

TABLE 4.

ANALYSIS OF COLUMN DATA.

Girder.		Length L m.	L/R.	Bending Moments at Failure (1). Horizontal. $M_h = Pd_h$ in. lb.	Vertical. $M_v = Pd_v$ in. lb.	Bending Stress at Failure (2). Horizontal. f_{bh} lb./in.²	Vertical. f_{bv} lb./in.²	Stresses at Failure (3). Average end. P/A lb./in.²	Total. $P/A + f_{bh} + f_{bv}$ lb./in.²
55	a	10	96	11440	640+	11810	260+	10000	22070+
	b	10	96	10720	910+	11070	370+	9060	20500+
	c	10	96	5980	90+	6180	40+	10060	16280+
71	a	5	72	10120	1690	10820	1920	14520	27260
	b	5	72	8950	1000	9570	1250	15780	26600
	c	5	72	8020	430	8570	530	15610	24710
72	a	5	72	6850	1100	7320	1460	16020	24800
	b	5	72	7720	1650	8250	2170	16490	26910
	c	5	72	8150	740	8710	1000	15610	25320
72	a	10	143	19060	770+	20370	600+	4710	25680+
	b	10	143	15950	560+	17050	440+	4860	22350+
	c	10	143	18860	1070+	20150	830+	5070	26050+
91	a	5	66	580+	530+	1210+	460+	19350	21020+
	b	5	66	1430	490	2980	430	15970	19380
	c	5	66	1260+	320+	2630+	280+	17760	20660+
92	a	5	66	2860	790	4900	570	17360	22830
	b	5	66	1110+	1200	1900+	850+	18090	20850+
	c	5	66	1550	1110	2650	790	17050	20490
92	a	10	132	9730	480+	16660	340+	5520	22520+
	b	10	132	7350	540+	12590	390+	5630	18610+
	c	10	132	7410	890+	12690	630+	5520	18840+
42	A-a	5	47	1640+	700+	1760+	440+	21990	24190+
	b	5	47	1680+	1680+	1800+	1050+	18770	21620+
	c	5	47	1210	600	1300	380	23780	25460
31	B-a	3	37	360+	1420+	610+	1410+	23990	26010+
	b	3	37	280+	450+	480+	450+	22870	23790+
	c	3	37	540	1010	920	1000	22010	23940
32	a	3	38	330+	1010+	570+	1000+	23340	24910+
	b	3	38	10+	1010+	20+	1000+	23470	24480+
	c	3	38	830+	1320+	1420+	1300+	21550	24270+

TABLE 4.

ANALYSIS OF COLUMN DATA.

Girder.	Length. L m.	L/R.	Maximum Average End Stress P/A max. lb./in.²	Vickers' Formula. Failing Stress in Channels P_v (4). Primary. lb./in.²	Secondary. lb./in.²	Average Stress at Failure. Johnson's Formula f_j (5a) lb./in.²	Euler's f_e (5b) lb./in.²
21 a	5	85	11410	—	—	11970	12370
b			12680				
c			10570				
23 a	5	85	11700	—	—	12430	12730
b			12210				
c			11320				
23 a	10	170	3620	—	—	Negative	3480
b			3340				
c			3380				
52 a	5	47	18620	—	—	17550	32630
b			18880				
c			17950				
53 a	5	49	17520	—	—	17640	31270
b			—				
c			17480				
54 a	5	47	15360	—	—	18160	33190
b			17220				
c			17880				
55 a	5	48	18180	18170(6)**	17020(6)**	17740	33860
b			16750	25570(7)	19670(7)		
c			19150	27370(8)	20420(8)		
55 A-1	5	48	16680	—	—	No	
2			17300			Beam	
3			16650			Tests.	
55 B-1	5	48	15510	—	—	No	
2			16590			Beam	
3			15060			Tests.	
55 C-1	5	48	17620	—	—	15650***	32350
55 D-1	5	48	23300	—	—	21660	36800
2			21940				

** For meaning of (6), (7) and (8) see notes on Analysis of Beam Tests.
*** Both 55 C beams failed in "shear." These values are therefore presumably too low.

TABLE 4.

ANALYSIS OF COLUMN DATA.

Girder.	Length L m.	L/R	Maximum Average End Stress P/A max. lb./in.²	Vickers' Formula. Failing Stress in Channels P_v (4) Primary. lb./in.²	Secondary. lb./in.²	Average Stress at Failure Johnson's Formula. f_j (5a) lb./in.²	Euler's f_e (5b) lb./in.²
55 a	10	96	10000	8570(6)**	8300(6)**	10150	10190
b			9930	9920(7)	8880(7)		
c			10610	10180(8)	9030(8)		
71 a	5	72	15280	—	—	16910	18140
b			16840				
c			16610				
72 a	5	72	16200	—	—	17080	17880
b			17310				
c			16310				
72 a	10	143	5070	—	—	Negative	5060
b			4860				
c			5070				
91 a	5	66	19350	14470(6)**	12720(6)**	16200	20070
b			15970	15830(7)	13620(7)		
c			17760	15930(8)	13720(8)		
92 a	5	66	17360	16570(6)**	13830(6)**	16350	19420
b			18090	18200(7)	14850(7)		
c			17050	18330(8)	14970(8)		
92 a	10	132	5630	5650(6)**	5290(6)**	460	5920
b			5670	5830(7)	5430(7)		
c			5630	5840(8)	5450(8)		
42 A-a	5	47	21990	—	—	No	
b			18770			Beam	
c			23780			Tests.	
31 B-a	3	37	23990	—	—	No	
b			22860			Beam	
c			22010			Tests.	
32 a	3	38	23340	—	—	No	
b			23470			Beam	
c			21550			Tests.	

** For meaning of (6), (7) and (8) see notes on Analysis of Beam Tests.

TABLE 4.

NOTES TO ANALYSIS OF COLUMN DATA.

(1) P = load in lbs.; d_h = lateral deflection in inches; d_v = vertical deflection in inches.

(2) $f_{bh} = M_h y_b / I_{min}$; where $M_h = P d_h$ and y_b and I_{min} are given in Table 1.

$f_{bv} = M_v y_c / I_{max}$; where $M_v = P d_v$, y_c and I_{max} are given in Table 1.

(3) In no case was the total stress, computed from the maximum load and the corresponding deflections, greater than that computed from the failing load and the corresponding deflections.

(4) Vickers' formula :—

$$1/P_v = 1/f_v + (L/R)^2/50{,}000$$

f_v is given in Table 4. This is taken for base channels because no failure of apex channel took place.

R is the least radius of gyration of columns.

(5a) A modified application of Johnson's formula :—

$$f_j = f_c \left[1 - \{ f_c (L/R)^2 / 4\pi^2 E \} \right]$$

f_j = average stress at failure, lb. per sq. in.

f_c = average total compressive stress of channel at failure, taken from beam data, Table 3.

E = average apparent Modulus of Elasticity in bending, taken from beam data, Table 3. This modulus was, however, determined for the axis of maximum moment of inertia.

The apparent modulus for the axis of minimum moment of inertia probably is higher and between this value and that determined by tension tests on the channels.

(5b) Euler's formula :—

$f_e = \pi^2 E/(L/R)^2 = P_e/A$ average end stress under Euler load.

Where E was taken from beam tests, Table 3.

Note that R is always used for radius of gyration of columns and r for radius of gyration of channels and angles.

TABLE 5.

ANALYSIS OF COMBINED BEAM AND COLUMN LOADING.

Girder.	Length. L m.	L/R.	Bending Moments at Failure. Beam load Vertical M_b (1) in. lbs.	Column Bending. Vertical $M_v = Pd_v$ (2) in. lbs.	Horizontal $M_h = Pd_h$ (3) in. lbs.
21 a	5	53.8	9850	2150+	80+
b		—	—	1830	500
c		—	—	1890	550
23 a	5	53.8	9910	1970+	460+
b		—	—	2440+	720+
c		—	—	2260+	840+
23 a	10	107.6	10620	1050+	50+
b		—	—	870	280
c		—	—	1000	360
52 a	5	30.0	18300	3480+	410+
b		—	—	3660+	1930+
c		—	—	3820+	2040+
53 a	5	29.5	18310	2830	630
b		—	—	2800	820
c		—	—	3110	1410
54 a	5	30.1	24490	4030	1190
b		—	—	4700	2780
c		—	—	4560+	1880+
55 a	5	29.6	24510	3540+	480+
b		—	—	3450	460
c		—	—	4880+	2090+
55 a	10	59.3	25280	7590	60+
b		—	—	5680	1910
c		—	—	5320	1830
71 a	5	52.1	14770	7050	370
b		—	—	6580	1060
c		—	—	6180	1410
72 a	5	52.1	14780	6150+	1570+
b		—	—	5700	950
c		—	—	3470	970
72 a	10	104.2	15600	8050	3820
b		—	—	6590+	1590+
c		—	—	8250	5380
91 a	5	42.0	9870	2750+	120+
b		—	—	2900+	620+
c		—	—	2680+	80+
92 a	5	42.0	9890	3490+	1080+
b		—	—	4270+	210+
c		—	—	3920+	1220+
92 a	10	84.0	10560	4290	3970
b		—	—	3910+	2060+
c		—	—	4910	2750

TABLE 5.

ANALYSIS OF COMBINED BEAM AND COLUMN LOADING.

Girder.		Length. L m.	L/R Max.	Maximum Average End Stress. P/A max. lbs./in.²	Stresses at Failure. Bending. Vertical. f_v (4) lbs./in.²	Bending. Horizontal. f_h (5) lbs./in.²	Average End Stress. P/A lbs./in.²	Total $P/A + f_v + f_h$ lbs./in.²	Failing Stress, Morley's f_{mv}(6) lbs./in.²
21	a	5	53.8	5540	13320+	220+	5540	19080+	18830
	b		—	4790	12960	1350	4790	19100	17700
	c		—	4700	13030	1480	4700	19210	17570
23	a	5	53.8	5400	13180+	1240+	5400	19820+	18640
	b		—	6410	13700+	1940+	6410	22050+	20060
	c		—	5680	13500+	2260+	5680	21440+	18960
23	a	10	107.6	990	12950+	130+	990	14070+	14210
	b		—	700	12750	750	700	14200	13440
	c		—	850	12890	970	700	14560**	13900
52	a	5	30.0	10690	10870+	520+	10690	22080+	21210
	b		—	9940	10960+	2450+	9940	23350+	20350
	c		—	10020	11040+	2590+	10020	23650+	20470
53	a	5	29.5	9450	10550	800	9450	20800	19800
	b		—	9260	10540	1040	9260	20840	19560
	c		—	9840	10690	1790	9840	22320	20160
54	a	5	30.1	8640	11590	1230	8400	21220	19780
	b		—	9090	11860	2870	9560	24290	21080
	c		—	9660	11810+	1940+	9660	23410	21090
55	a	5	29.6	11360	11400+	500+	11360	23260+	22790
	b		—	11200	11370	480	11200	23050	22520
	c		—	10940	11950+	2160+	10940	25050+	22250
55	a	10	59.3	6000	13360	60+	6000	19420+	19060
	b		—	4710	12590	1970	4710	19270	17020
	c		—	4510	12440	1890	4510	18840	16670
71	a	5	52.1	9660	17000	400	9510	26910	26060
	b		—	9720	16630	1130	9720	27480	26180
	c		—	9250	16320	1510	9250	27080	25400
72	a	5	52.1	9660	16310+	1680+	9660	27650+	26150
	b		—	9130	15960	1020	9130	26110	25270
	c		—	6420	14220	1040	5770	21030**	20980
72	a	10	104.2	3030	18420	4080	3030	25530	21090
	b		—	2740	17290+	1800+	2740	21830+	20030
	c		—	2830	18580	5750	2830	27160	20240
91	a	5	42.0	9630	10970+	250+	9630	20850+	20280
	b		—	10470	11100+	1290+	10470	22860+	21330
	c		—	9350	10910+	170+	9350	20430+	19980
92	a	5	42.0	11650	9540+	1850+	11650	23040+	20970
	b		—	12690	10100+	360+	12690	23150+	22320
	c		—	12000	9850+	2090+	12000	23940+	21420
92	a	10	84.0	3630	10590	6800	3630	21020	13670
	b		—	3320	10320+	3530+	3320	17170+	13090
	c		—	3900	11030	4710	3700	19440	14210

** These girders show slightly lower stresses at failure than at maximum load. The differences (150 and 250 lb./in.²) are within the experimental error.

TABLE 5.

ANALYSIS OF COMBINED BEAM AND COLUMN LOADING.

NOTES.

(1) $M_b = 0.12931\,LW + 0.125\,Lw$ (derivation explained in Table 3)

where W = total vertical load (transverse load).

w = weight of the girder.

(2) $M_v = Pd_v$

where P = end load at failure.

d_v = vertical deflection at centre of girder.

(3) $M_h = Pd_h$

where P = end load at failure.

d_h = horizontal deflection at centre of girder.

For values of P, d_v and d_h see Table 2.

(4) $f_v = (M_v + M_b)\, y_c / I_{max}$.

I_{max} and y_c are given in Table 1.

(5) $f_h = M_h y_b / I_{min}$.

I_{min} and y_b are given in Table 1.

(6) $f_{mv} = M' y_c / I_{max} + P/A$ = total failing stress using Morley's formula.

Where M' = Morley's formula for bending moment from combined transverse and end loads.

$$= (8M_b E I_{max} / PL^2)\,(\sec \tfrac{1}{2}\pi \sqrt{P/P_e} - 1)$$

M_b is given above.

E = average apparent modulus of elasticity from beam tests.

P = end load at failure.

$P_e = \pi^2 E I_{max} / L^2$ Euler's limiting value for the ideal column.

L = length of girder.

Note that as applied above only the one deflection is considered. As will be seen the effect of the lateral deflections d_h is considerable. If the end load stress (P/A) and the stress due to vertical deflection (f_v) be added and the sum compared with the stress computed by the above method it will be found that they are in very close agreement. This is what might be expected when it is remembered that the girders were initially very straight and failure always occurred at stresses below the elastic limit.

TABLE 6.

COMPARISON OF COMPRESSIVE STRESSES IN CHANNELS AND ANGLES FROM INDIVIDUAL GIRDER TESTS.

Girders.				Channel or angle on compression side.		Pitch of lattices on channels (see Fig. 7).	
Type.	Sec. Area. A in.².	R in.	L/R	Mark.	r_0 in.	l_1 in.	l_2 in.
Girders of triangular section, tested as 196.8 in. (5 metres) lengths.							
21	.2129	2.31	85	B-1	.194	7.875	5.83
23A	.2129	2.31	85	B-1	.194	3.9375	—
42A	.2733	4.182	47	B-3	.188	7.875	5.83
52	.2395	4.185	47	B-2	.192	11.811	9.72
53	.2592	4.023	49	B-2	.192	11.811	9.72
54	.2930	4.191	47	B-3	.188	11.811	9.72
55	.3081	4.087	48	B-3	.188	11.811	9.72
55A	.3081	4.087	48	B-3	.188	11.811	9.72
55B	.3081	4.087	48	B-3	.188	11.811	9.72
55C	.3081	4.087	48	B-3	.188	11.811	9.72
55D	.3081	4.087	48	B-3	.188	5.905	—
91	.2129	2.976	66	B-1	.194	7.875	5.83
92	.2592	2.977	66	B-2	.192	7.875	5.83
Girders of triangular section, tested as 393.7 in. (10 metres) lengths.							
23A	.2129	2.31	170	B-1	.194	3.9375	—
55	.3081	4.087	96	B-3	.188	11.811	9.72
92	.2592	2.977	132	B-2	.192	7.875	5.83
Girders of triangular section, tested as 118.11 in. (3 metres) lengths.							
31B	.2226	3.21	37	B-2	.192	7.875	5.83
32	.2395	3.097	38	B-2	.192	7.875	5.83
Girders of rectangular section, tested as 196.8 in. (5 metres) lengths.							
71	.3396	2.754	72	N-3	.183	7.875	5.83
72	.3396	2.754	72	N-3	.183	5.75	5.83
Girders of rectangular section, tested as 393.7 in. (10 metres) lengths.							
72	.3396	2.754	143	N-3	.183	5.75	5.83

TABLE 6.

COMPARISON OF COMPRESSIVE STRESSES IN CHANNELS AND ANGLES FROM INDIVIDUAL GIRDER TESTS.

Girders. Type.	L/R	"l/r" corresponding to various pitches. l_1/r_0.	l_2/r_0.	Corresponding stresses from curves of Fig. 8. f_1 lbs./in.^{2}	f_2 lbs./in.^{2}
Girders of triangular section, tested as 196.8 in. (5 metres) lengths.					
21	85	40.6	30.0	21,500	25,000
23A	85	40.6	30.0	21,500	25,000
42A	47	41.9	31.0	29,000	32,500
52	47	61.5	50.7	18,000	22,000
53	49	61.5	50.7	18,000	22,000
54	47	62.8	51.7	16,000	22,500
55	48	62.8	51.7	16,000	22,500
55A	48	62.8	51.7	16,000	22,500
55B	48	62.8	51.7	16,000	22,500
55C	48	62.8	51.7	16,000	22,500
55D	48	31.4	—	36,000	—
91	66	40.6	30.0	21,500	25,000
92	66	41.0	30.4	25,000	28,500
Girders of triangular section, tested as 393.7 in. (10 metres) lengths.					
23A	170	40.6	30.0	21,500	—
55	96	62.8	51.7	16,000	22,500
92	132	40.6	30.0	25,000	28,500
Girders of triangular section, tested as 118.11 in. (3 metres) lengths.					
31B	37	41.0	30.4	25,000	28,500
32	38	41.0	30.4	25,000	28,500
Girders of rectangular section, tested as 196.8 in. (5 metres) lengths.					
71	72	43.0	31.9	26,000	31,000
72	72	31.4	31.4	31,000	31,000
Girders of rectangular section, tested as 393.7 in. (10 metres) lengths.					
72	143	31.4	31.4	31,000	31,000

TABLE 6.

COMPARISON OF COMPRESSIVE STRESSES IN CHANNELS AND ANGLES FROM INDIVIDUAL GIRDER TESTS.

Type.	Mean ultimate stresses computed from tests as Beams. lb./in.²	Columns. lb./in.²	Combined. lb./in.²	Mean E from Beam Test. lb./in.²	Direct Stress Column Test P/A. lbs./in.²	Col. Stress, Johnson's Formula. f_j (8). lbs./in.²	Note.
Girders of triangular section, tested as 196.8 in. (5 metres) lengths.							
21	20,300	19,050	19,130	9,053,000	11,550	11,970	
23A	21,586	20,270	21,100	9,318,000	11,740	12,430	9
42A	—	23,760	—	—	21,510	—	
52	20,896	19,320	23,030	7,303,000	18,480	17,550	
53	21,593	20,060	21,320	7,607,000	17,500	17,640	
54	21,700	18,820	22,970	7,426,000	16,820	18,160	
55	21,240	19,790	23,790	7,903,000	18,030	17,740	
55A	—	19,400	—	—	16,880	—	10
55B	—	16,400	—	—	15,720	—	11
55C	18,220	18,600	—	7,552,000	17,620	15,650+	12
55D	26,400	25,690	—	8,590,000	22,620	21,660	13
91	22,510	20,350	21,380	8,858,000	17,690	16,200	
92	23,330	21,390	23,380	8,571,000	17,500	16,350	
Girders of triangular section, tested as 393.7 in. (10 metres) lengths.							
23A	21,390	17,410	14,280	10,206,000	3,450	Negative	
55	21,643	19,620	19,180	9,517,000	10,180	10,150	
92	23,200	19,990	19,210	10,448,000	5,640	460	
Girders of triangular section, tested as 118.11 in. (3 metres) lengths.							
31B	—	24,580	—	—	22,950	—	
32	—	24,550	—	—	22,790	—	
Girders of rectangular section, tested as 196.8 in. (5 metres) lengths.							
71	27,076	26,190	27,160	9,529,000	16,240	16,910	
72	28,170	25,680	24,930	9,393,000	16,610	17,080	
Girders of rectangular section, tested as 393.7 in. (10 metres) lengths.							
72	27,420	24,690	24,840	10,478,000	5,330	Negative	

TABLE 6.

COMPARISON OF COMPRESSIVE STRESSES IN CHANNELS AND ANGLES FROM INDIVIDUAL GIRDER TESTS.

NOTES.

1. This table is intended to show the relation between the mean ultimate compressive stresses in the channels and angles as found from tests of the girders at the Bureau of Standards, and the same stresses as found from tests of the sections at the Mass. Inst. of Tech.

2. All lattices are .019 in. thick except those on sides of types 32, 55, 55A and 55B which are .023 in. thick.

3. For derivation of l_1 and l_2, see Figs. 3 to 7 inclusive.

4. All dimensions are in inches.

5. All forces in lbs.

6. Sectional areas are from computed values given in Fig. 2.

7. "r_0" of channels are from values computed by M.I.T. from actual sections by enlargement. They are slightly smaller than those which will be found from data of Fig. 2.

8. Johnson's column formula for average compressive stress at failure—f_j.

$$f_j = f_c - b\,(L/R)^2$$

where $b = f_c^2/4\pi^2 E$

f_c = compressive stress at failure from column test.

E = modulus of elasticity from beam test.

9. Type 23 girder is the same as type 21 except the pitching of the lattice crosses on the sides. The crosses on 21 are at 7.875 in. pitch; on 23 they are at half this or 3.9375 in. pitch.

10. Type 55-A girder is the same as the standard type 55, except that an irregular lattice spacing occurring at the middle—and caused by a joint which should have come there—was done away with and the lattice pitching equalised.

11. Type 55-B girder is the same as the standard type 55, except that the lattices are not spiralled around the girder. The resulting girder is most like the conventional structural column of angles and lattice bars.

12. Type 55-C girder is the same as type 55-B, but with the lightest lattices available to bring out the influence of the lattices on the strength of the member. Unfortunately the change in thickness is very small.

13. Type 55-D girder is the same as type 55-C, but with the lattice cross-pitch cut in half. Unfortunately the extremely light lattices, which should have been used in this girder, were not available. Due to the overlapping crosses the maximum length of unsupported channel becomes 3.815 in.

SECTION II.

Static Forces Affecting the Longitudinal Strength

In an airship floating in static equilibrium in the atmosphere there are inevitable inequalities in the longitudinal distribution of weight and buoyancy, producing what are usually designated "the primary static shearing forces and bending moments." The computations of the distribution of the weight and buoyancy along the hull, and the resulting shearing forces and bending moments according to the well known methods of naval architecture, present no difficulties; and the procedure to be followed in such computations calls for no explanation, except to note that the weight and buoyancy forces are most conveniently assumed to be concentrated at the main transverse frames, instead of distributed continuously along the hull. This assumption is justified because the buoyancy forces are carried to the main frames by the wiring and the longitudinal girders; and the principal loads are also carried to these frames by the wiring and the keel and the car suspension.

The designer endeavours to dispose the loads in the manner most suitable for the structure, in so far as other considerations may permit; but it should be remembered that in a modern rigid airship the disposable and movable loads, such as the fuel, ballast, passengers and crew, constitute such a large proportion of the total weight of the airship that improper distribution of these loads while in flight may easily bring about dangerously large stresses. It is the duty of the designer to determine a limiting range for the infinite variety of possible load distributions in all conditions of fullness of gas cells, or altitude of the airship, such that the static shearing forces and bending moments will not become excessive. It is the duty of the commander of the airship to maintain the actual load distribution within the limits set by the designer.

Curves of the loads, shearing forces and bending moments for the airship ZR.1 in some typical conditions are given in Fig. 15. An interesting point to be learned from these curves is that the maximum bending moment in the full load condition is increased from 97,450 metre-pounds to 196,310 metre-pounds by the following small changes in the distribution of the loads:—

2,200	lbs.	of	ballast	from	frame	No.	100 to frame No.	40.
2,200	,,	,,	,,	,,	,,	110	,,	170.
1,220	,,	,,	,,	,,	,,	70	,,	60.
610	,,	,,	,,	,,	,,	70	,,	80.
1,220	,,	,,	,,	,,	,,	130	,,	160.

The change in the bending moment due to this movement of the disposable loads causes an increase of about 2,200 lbs./in.^{2} in the maximum tensile and compressive stresses in the longitudinals.

In the event of a gas cell becoming deflated, it is necessary to maintain static equilibrium of weight and buoyancy by dropping fuel or ballast or other weights. It may also be necessary partially to deflate the adjacent gas cells in order to relieve the pressure on the transverse frames, and this additional loss of gas will require further jettisoning of disposable loads. So far as possible, the loads should be dropped from the region of the loss of lift, but even with this precaution, large static shearing forces and bending moments may be developed. The proper distribution of loads and the magnitudes of the resulting shearing forces and bending moments in various deflated gas cell conditions must be investigated by the designer; but the problem is quite simple and straightforward, although it may be somewhat long and laborious. It was found that in the ZR.1, in the fully loaded condition, the complete deflation of the gas cell between frames 110 and 120, and half deflation of the adjacent cells in order to relieve the pressure on these frames, and with disposable loads jettisoned in the most favourable

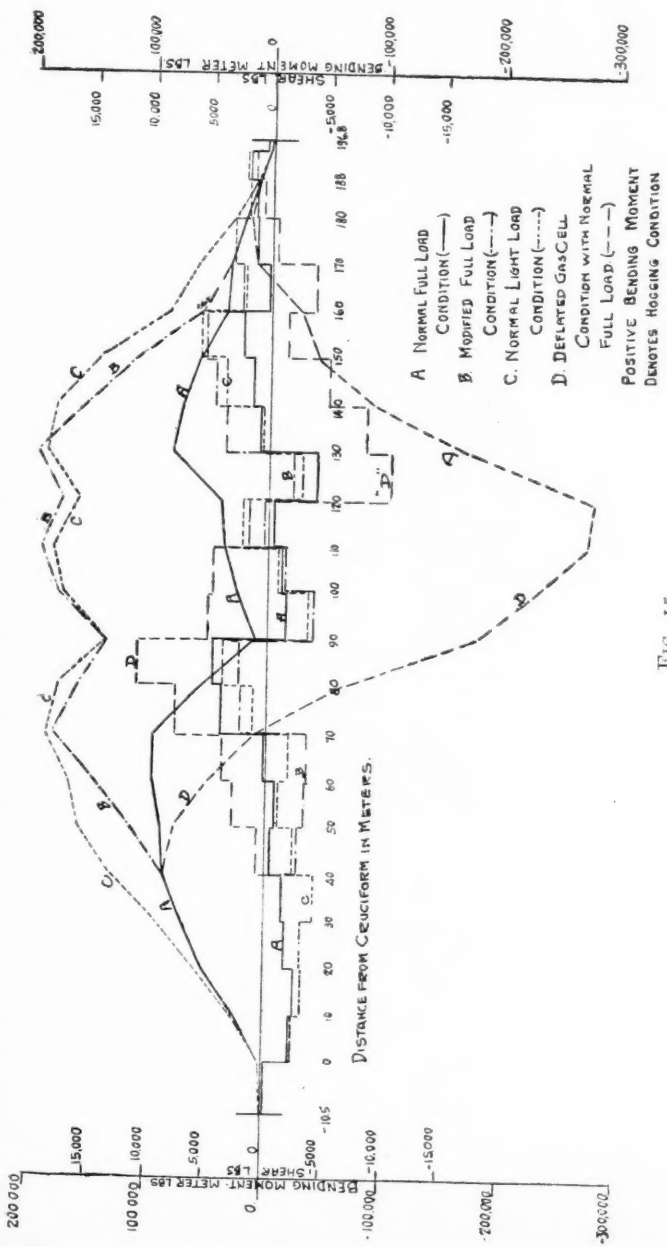

SHEAR LBS
BENDING MOMENT METER LBS
DISTANCE FROM CRUCIFORM IN METERS.
A NORMAL FULL LOAD CONDITION(——)
B MODIFIED FULL LOAD CONDITION(— —)
C. NORMAL LIGHT LOAD CONDITION(·····)
D. DEFLATED GAS CELL CONDITION WITH NORMAL FULL LOAD (— — —)
POSITIVE BENDING MOMENT
DENOTES HOGGING CONDITION

FIG. 15.

Curves of static shear and bending moment for airship ZR.1.

manner possible, the bending moment at frame 120 is changed by 358,360 metre-pounds from 81,120 metre-pounds hogging, to 277,240 metre-pounds, sagging. The corresponding change in stress in the top longitudinal is about 7,900 lbs./in.^{2}. It may be noted for comparison that the largest calculated bending moment from aerodynamic forces on the ZR.1 will be shown to be 318,000 metre-pounds, so that a deflated gas cell can cause severe stresses and great care must be exercised in operating under such conditions.

A static bending moment is also produced by the increase of gas pressure upwards in the airship. The magnitude of this bending moment is given by the well known formula:—

$$M = \tfrac{1}{4}\pi k R^4,$$

where M = bending moment.

k = lift of gas per unit volume.

R = radius of the cross-section of gas cell where the bending moment is desired.

This bending moment is of importance in non-rigid airships where the diameter is relatively large in proportion to the length. In the rigid airship ZR.1, inflated with gas lifting .068 lbs./ft.^{3}, the maximum bending moment from gas pressure is only 36,000 metre-pounds, a small moment compared with the possible bending moments from other sources.

In airships having gas cell netting composed of diagonal wires the longitudinal forces, including the bending moment, from the gas pressure are almost completely balanced by the longitudinal components of the tensions in the netting. The authors have calculated that in the R.38, where the gas cells were held by circumferential wires with catenary loops to take the lift, there was an important tension produced in the longitudinals by the gas pressure. For example, if the cells were inflated with gas lifting .068 lbs./ft.^{3}, with 10 mm. of water super-pressure, the longitudinal force and the bending moment due to the gas pressure would produce a tension of 3,339 lbs./in.^{2} in the top longitudinal, diminishing to 1,618 lbs./in.^{2} in the bottom longitudinal. Since the girders are about twice as strong in tension as in compression the addition of a longitudinal tensile force will usually increase the ultimate strength of the hull. The calculation points to the superiority of circumferential wires over diagonal wire netting; but, on the other hand, the diagonal wires assist materially against shear, and if they are omitted the main and secondary shear wire systems should be correspondingly strengthened.

Section III.

Aerodynamic Forces Affecting Longitudinal Strength

Aerodynamic Forces

In addition to the vertical static forces of weight and buoyancy, there may be transverse forces, either vertical or horizontal, resulting from differences in the air pressure on the opposite sides of the fins and control surfaces, or from unsymmetrical distribution of the air pressure around the cross-sections of the hull.

In the early rigid airships, constructed in Germany and England, the speed was low and the principal weights were concentrated at or near two or three power cars. Consequently the shearing forces and bending moments, due to the static forces of weight and buoyancy, greatly exceeded those due to the aerodynamic forces on the hull and tail surfaces; and if the hull structure had a fair margin of strength against static forces, the aerodynamic forces might safely

be neglected. As airships became larger and faster and with more power cars, the weights were more evenly disposed longitudinally, so the static forces became relatively less serious, while the aerodynamic forces, being proportional to the square of the speed, rose rapidly in magnitude and importance.

For the purpose of our investigation, five principal conditions of aerodynamic forces will be considered, as follows:—

(*a*) Sudden application of the rudders or elevators when the airship is proceeding along a straight path.

(*b*) The airship proceeding along a straight horizontal path at a fixed angle of pitch to compensate for excess of weight over buoyancy, or vice versa.

(*c*) Steady circling flight.

(*d*) Sudden changes in the direction of the wind when the airship is in straight flight.

(*e*) Lying to a mooring mast.

Other possible aerodynamic conditions may be considered as combinations of these five.

Effect of Sudden Application of the Rudders and Elevators

Several investigators, working independently in the early stages of the development of the problem of aerodynamic forces, came to the conclusion that the most severe aerodynamic condition occurs when the rudders or elevators are first put over. It was urged that this is the worst condition because a transverse force applied to the hull through the centre of pressure on the tail surfaces produces a turning moment opposed by the rotational inertia of the airship; and as the angular motion increases, the swinging of the stern reduces the force produced on the tail surfaces by the rudders or elevators. A method of procedure developed by the authors for calculation of the inertia forces and the resulting shearing forces and bending moments when the rudders are suddenly put over is described in Appendix II.

A method of procedure has also been developed by Professor L. Bairstow and is described in R. and M. No. 794. Professor Bairstow regards the motion as a rotation about the centre of gravity combined with a lateral translation of that centre. The combined motion is equivalent to a pure rotation about the centre of percussion.

The authors have also investigated the rate of angular acceleration which would be produced by the pressure on the rudders if suddenly thrown to 20° when the airship is proceeding at full speed. It is shown by the calculation described in Appendix II. that the maximum bending moment for ZR.1 would be 107,000 metre-pounds, and the angular acceleration 26°/sec.^{2}. This rate of angular acceleration is so large that the condition of sudden application of the rudders at 20°, when the airship is in straight flight, could never occur in actual operation, because the airship will respond to the movement of the control surfaces before they reach their full throw. It follows that the control surfaces and their operating mechanisms should be designed (and have been for ZR.1) to prevent too rapid movement of their surfaces, in order to obviate the large forces and bending moments which might be produced by the surfaces reaching full throw before the stern swings appreciably.

Subsequent investigations, to be described later in this paper, showed that when the airship reaches the condition of steady turning, the resultant force on the tail surfaces is in the opposite direction to that produced by the initial movement of the rudders; and there is also a transverse aerodynamic force on the hull, largely concentrated in the region of the bow, and these two sets of forces

acting towards the centre of the turning circle give equilibrium with the centrifugal forces. Owing to the concentration of the inward acting forces near the bow and the stern, opposed by the centrifugal forces distributed along the hull, there is a bending moment of opposite sign to that produced by the initial application of the rudders and possibly of greater magnitude. The calculation of this bending moment will be considered in the section on the aerodynamic forces in a steady turn.

The calculation of the stresses resulting from sudden application of the rudders is evidently quite arbitrary and takes a view of actual conditions that cannot be justified. It is included here only for its value as a powerful means for comparing the hull strengths of different airships, for which purpose it has already been found useful. The actual stresses in flight produced by rudder action will only be learned from full-scale experiments, but it is evident that such stresses may be high if the helm is roughly handled. There is ample reason to urge that an airship in an unfavourable condition as to static or dynamic loading be handled with care and preferably at reduced speed.

Aerodynamic Forces at a Fixed Angle of Pitch

When an airship is flown either " light " or " heavy," use is made of aerodynamic forces to balance an excess of buoyancy over weight or the reverse. The ship is flown with longitudinal axis inclined in the vertical plane at an angle to the direction of motion, and the ship may be regarded as receiving the relative wind at a fixed angle of pitch, with the elevators set at an appropriate angle to give equilibrium (see Fig. 16).

In this unsymmetrical attitude the aerodynamic pressures on the hull of the ship are distributed in such a manner as to produce a resultant couple tending to upset the ship, and a resultant force resolving into a drag greater than the drag for the normal attitude, and a dynamic lift which adds to the force of buoyancy due to gas displacement. Balancing these forces we have, of course, the weight of the airship, the thrust of her propellers and the aerodynamic force on the fins and elevators. This last named force, F, is adjusted by the pilot by setting the elevators until equilibrium of forces and moments is obtained and the ship proceeds in level flight, but with a fixed angle of pitch. The inherent statical righting moment due to the position of the centre of gravity below the centre of buoyancy assists the balancing force to maintain the airship in equilibrium, and the statical righting moment may be increased by the pilot if he chooses to shift weights, *e.g.*, by moving members of the crew along the ship. It should be noted, however, that since the dynamic lift is due mainly to F, any action taken to reduce F will also reduce the dynamic lift at a given speed and pitch angle.

The use of dynamic lift to operate an airship below her pressure height is a fundamental of operation and must be considered a normal condition for which the strength of the ship is to be calculated. Bending moments imposed on the hull by the distribution of aerodynamic forces on hull and fins are quite independent of the statical bending moments previously discussed and are, in fact, to be considered as superimposed thereon. It will be found in a high-powered airship that aerodynamic bending moments are more serious than the most unfavourable case of statical bending. To devise for this condition of flight a practical, if approximate, method of strength calculation is therefore most essential.

In order to compute the aerodynamic bending moment imposed on the hull at some fixed angle of trim, it is necessary to know three things:—(1) The speed of the airship; (2) the distribution of aerodynamic pressure along the hull; and (3) the force on fins and elevators necessary for equilibrium. This required information may be obtained by wind tunnel experiments with a model, by hydrodynamic calculation, or from flight experiments with actual airships. Lacking data from full-scale experiments, the last named source is hardly helpful to-day,

MOMENT OF AIR PRESSURE FORCES ON THE BARE HULL
RESULTANT OF BUOYANCY FORCES
C.B.
EXCESS OF WEIGHT OVER BUOYANCY
C.G.
UPWARD AIR PRESSURE
MOMENT OF STATIC FORCES OF WEIGHT AND BUOYANCY
RESULTANT OF WEIGHT FORCES
DOWNWARD AIR PRESSURE
UPWARD PRESSURE ON TAIL SURFACES

FIG. 16.

Illustrating equilibrium between weight, buoyancy and air pressure forces on hull and tail surfaces.

although it will be shown that such dynamic lift and turning trial data as are available indicate that neither the wind tunnel nor the calculated results can be greatly in error.

As part of the investigation into the loss of R.38, the National Physical Laboratory, Teddington, determined the distribution of normal air pressure over the surface of a model of the airship L.33, when held in the wind tunnel at a fixed angle of pitch. At the same time the aerodynamic moment, lift and drag, were measured for the model as a whole with and without fins. Such a pressure plotting investigation is elaborate and laborious, but if carefully done, gives a very complete picture of conditions.

From the observed increase of drag with angle of pitch, speed power curves for the airship can be constructed from which for any angle of pitch the maximum horizontal speed possible with the engines fitted can be read off. It appears fortunate that high speed is impossible at any considerable angle of pitch as this forced slowing down very greatly relieves the hull of the heavy bending moments that are associated with high speed.

From the wind tunnel observations of normal aerodynamic pressures on the model, a curve of distribution of force normal to the axis of the hull is made. The force on the fins and elevators necessary to balance the ship in pitch may be determined from a series of wind tunnel tests with different elevator settings, or may be estimated with allowance for the inherent statical righting moment. Knowing the angle of pitch, speed of airship, distribution of static loads and air pressures, and the balancing force, the computation of the resultant bending moments on the hull is straightforward.

The greatest inconvenience in the procedure outlined above is the extensive wind tunnel investigation required. For preliminary design studies, for comparisons between existing ships and proposed designs and in cases where a direct method of computation is required, wind tunnel investigation is impracticable.

A highly ingenious, if somewhat artificial, theoretical treatment of the problem has been devised by Dr. M. M. Munk and further discussed by Dr. L. B. Tuckerman (N.A.C.A. Technical Reports Nos. 104 and 129). The procedure, in brief, is to compute the distribution of pressure along the hull by hydrodynamic theory, assuming ideal streamline flow in a perfect fluid, and then to apply quite arbitrarily to the fins and rudders the necessary force to preserve equilibrium of moments. Equilibrium of vertical forces is assumed to be taken care of by some difference between weight and buoyancy. The bending of the hull is computed just as when using wind tunnel data for the distribution of force on the bare hull (see Appendix III.).

The method is frankly approximate. Certain approximations are needed to effect the solution of the hydrodynamical equations and the assumption of a perfect fluid ignores the resultant dynamic lift in a real fluid. In reality, the streamlines in air do not close in so smoothly over the stern and hence pressures on the after body are less than computed. To balance the effect of this, we find that the assumed force on the tail necessary to balance the pitching moment is required to be larger than where wind tunnel data are used.

The most comprehensive data known to the authors on dynamic lift trials of a modern rigid airship was found in the notebook of an officer who lost his life in the destruction of the German airship L.70 over the east coast of England in September, 1918. The data was for an airship of the Zeppelin 56,000 cubic metre class, most probably the L.41. The trials were carried out at an altitude of 6,000 metres, at various angles of pitch up to 12 degrees, and with from one to five engines in operation. This notebook was made available to a committee formed by the Air Ministry following the R.38 investigation, and an analysis of the information therein contained shows that irrespective of the number of engines

in operation, and at all angles of pitch up to 12 degrees, the variation of speed with pitch angle is given very closely by the expression—

$$V = V_0 (1.c^{\alpha} .),$$

where V = speed at angle of pitch α, expre in degrees.
V_0 = speed at zero pitch.

In calculating the aerodynamic forces in the ZR.1, it was assumed that the speed varied with the angle of pitch according to this formula.

Comparative calculations of the dynamic lift of the ZR.1 have been made on three different bases:—(*a*) The N.P.L. pressure distribution curves; (*b*) the Munk formula; (*c*) the dynamic lift trials of the L.41. The calculations were all made for the standard atmosphere at 6,000 ft. altitude. The results are shown in the following table:—

Calculations of the Dynamic Lift of the Airship ZR.1 at 6,000 ft. Altitude.

Angle of pitch	6°	12°
Speed, ft./sec.	68	48.5
Dynamic lift, lbs.:		
(*a*) By N.P.L. pressure distribution on R.33 ...	7260	8120
(*b*) By Munk's formula on ZR.1	6500	6500
(*c*) By L.41 trials	6000	7850

Except at large angle of pitch the results of calculation and model and full-scale experiments for dynamic lift are in surprisingly close agreement. The dynamic lift for the L.41 trials at 6 degrees is seen to be lower than that calculated, as must be expected in view of the inclusion in practice of a righting moment due to the airship's statical stability. It is also probable that on the trials men were moved forward to help balance the ship by the shift in weight. Captain Heinen states it is German practice to move men forward until the ship is at proper trim with elevators neutral. It is also shown by comparative calculations in Appendix III. that the discrepancy between the observed and the theoretical distribution of transverse force along the after-body makes very little difference in the resulting bending moments.

For 6 degrees pitch, the maximum aerodynamic bending moment on ZR.1 is 2.5 per cent. less by Munk's formula than when calculated by the N.P.L. curve of transverse force. At 12 degrees pitch, the practical limit, it is 7 per cent. less by Munk's formula. At larger angles, neither Munk's formula nor high speed flight are practicable.

Aerodynamic Forces in a Steady Turn

Dr. Munk's theory of aerodynamic forces, with an amendment by Dr. Tuckerman, includes the case of steady turning of the airship (see N.A.C.A. Technical Reports, Nos. 106 and 129). The treatment is similar in principle to the case of fixed angle of yaw or pitch, but is very much complicated by the effect of rotation combined with an angle of yaw. It is shown by Dr. Tuckerman that the aerodynamic forces are distributed along the hull, according to the equations:

(*a*) At the bow outward.

$$F = (\rho V^2 Q/2R)\,(k_1 + D^2/2L^2)$$

(*b*) At the centre of pressure on the tail surfaces *inward*.

$$F = (\rho V^2/R)\,Q\,(1 + k_1 + k_2 \sin^2 \alpha)$$

(*c*) At the stern *outward*.

$$F = (\rho V^2 Q/2R)\,(k_1 + D^2/2L)$$

(*d*) A force distributed along the hull *outward*.

$$dF/dx = f = (\rho V^2/R)\,[\,\{\,\alpha(1 + k_1 + k_2 \sin^2 \alpha) - k_2 x\,\}\,dS/dx + (1 + k_2 + k_2 \sin^2 \alpha - D^2/2L^2)\,S\,]$$

The symbols have the same significance as in the previous case, with the exception that α is the angle of yaw at the centre of buoyancy only, defined as the angle between the longitudinal axis of the airship, and the trajectory of the centre of buoyancy. Additional symbols are:

a = distance from the centre of buoyancy to the centre of pressure on the tail surfaces.
R = radius of the turning circle.
Q = air displacement of the airship.

According to Tuckerman's extension of Munk's theory, the relation between the radius of the turning circle and the angle of yaw at the centre of buoyancy is given by

$$\sin 2\alpha = (2a/R)\ \{\ (1+k_1+k_2 \sin^2 \alpha)/(k_2-k_1)\ \}$$

The concentrated forces (a) and (c) at the bow and stern are small, but owing to their positions they contribute about 5 per cent. to the total bending moment in a steady turn.

There is at present no available experimental data with which the theoretical aerodynamic forces in a steady turn can be compared; but the good experimental confirmation of Dr. Munk's theory of the distribution of forces at a fixed angle of pitch gives confidence that the theory of the forces in a steady turn is not seriously in error. A check upon the theory may be obtained by comparison of the theoretical with the observed relations between the radius of the turning circle, the distance of the centre of pressure on the tail surfaces from the centre of buoyancy, and the angle of yaw at the centre of buoyancy. The researches of the N.P.L. (R. and M., Nos. 716 and 749) into the turning circles of British rigid airships showed that this relation may be expressed by the formula:—

$$R\alpha = .9\,a$$

where the symbols have the same significance as in Dr. Munk's formulæ. Taking a numerical example, for the airship ZR.1, when $R = 2{,}114$ ft. and $a = 302$ ft., it is found by the Munk formula that $\alpha = 9^\circ\ 31'$, whereas by the N.P.L. formula it is only $7^\circ\ 21'$. The discrepancy seems reasonable in view of the absence of a resultant inward acting force due to the angle of yaw in the ideal fluid. In the real fluid, this force reduces the angle of yaw necessary to give equilibrium with the centrifugal force, and should slightly reduce the bending moment calculated from Munk's theoretical forces.

The bending moments in the ZR.1, when proceeding in a steady turn of 2,114 ft. radius at 85 ft./sec., have been calculated by Munk's formulæ, and the maximum was found to be 230,000 metre-pounds.

Aerodynamic Forces Due to Weather

It is obvious that any airship has certain limitations upon its operation placed by wind and weather. Airships are somewhat in the position of the modern steamship which is forced to slow down in a heavy seaway to preserve her structural integrity. Consequently, a rigid airship being of great size and delicate construction should be operated with due consideration for the weather. The most effective way to maintain a high factor of safety is to operate the ship at reduced speed when at low altitudes. This has an important effect in relieving the aerodynamic loads to which the ship may be subjected when turning and manœuvring, regardless of weather conditions.

An additional advantage in reserving full speed for fair altitudes is to be found in relatively smoother air at such levels. The turbulence of air flowing over rough ground, which is felt as gustiness, is too highly complex to lend itself to exact measurement or general definition. The altitude at which the gusts are no longer felt depends upon the nature of the country and the season of the

year, but ordinarily above 2,000 feet conditions should be calm, although in the winter, at a time of high pressure, gusts have been felt at altitudes above 4,000 feet. We have concluded to omit a strength calculation for the case of a ship running at full speed through a gusty surface wind.

In addition to the general turbulence of the wind at ground level, there are to be expected large eddies caused by prominent obstructions, such as a hill or a cliff, and these eddies may be felt at an altitude of from four to five times the height of the obstruction. If the ship is restricted to full speed at a good altitude, the effect of such eddies is not of importance.

Convection currents due to unequal heating of air near the ground give rise to vertical movement in the air which extends to considerable altitudes. Professor Humphreys, on page 215 of his "Physics of the Air," states that pilot balloon measurements have shown vertical velocities of more than three metres per second either up or down under conditions not necessarily associated with the formation of clouds. This must be considered as an ordinary condition which the ship will encounter without warning when running at full speed.

The most serious condition of vertical air velocity is to be met with in and around cumulus clouds. Mr. C. F. Brooks, in the "Monthly Weather Review" for June, 1922, describes observations made by him of the vertical motion of cumulus tops, in which he observed vertical motions of 3, 4 and 7 metres per second. Professor Humphreys, in his "Physics of the Air," page 105, estimates that in order to account for the hailstones that are often formed in cumulus clouds, there must be an uprush of air of the order of 8 to 10 metres per second.

From a general survey of existing meteorological data, it seems clear that the winds inside a well-developed cumulus cloud are of great violence and present a real menace to an airship. In addition to a possible and probable vertical current of more than 7 metres per second inside the cloud, there may be a down draught around the outside edges of the order of 1 to 2 metres per second due to cooling from evaporation in this region. This would indicate that there are fairly narrow boundaries between air that is affected by the conditions causing the cumulus cloud and relatively undisturbed air, and an airship running into such a cloud will experience a sudden and violent force. This force it seems entirely impossible to calculate, and cumulus clouds should be considered a warning of danger and should be avoided.

It is possible that a ship under certain atmospheric conditions may run into disturbed air, possibly unmarked by clouds, at the boundary between two strata of unequal temperature, and in this condition may encounter a peculiar wind structure. There is evidence from windrow cloud formations of a relatively slow period wave motion at such a "sheet" of discontinuity similar to deep sea rollers. The nature of these waves has not been determined quantitatively, but according to Helmholtz's theory, the wave length L is given by an expression of the form

$$d_1 (u-V)^2 + d_2 (V-v)^2 = \tfrac{1}{2} g L (d_2 - d_1)$$

where V is the velocity of wave propagation, and d_1 and d_2 the densities of the layers whose velocities are u and v. For plausible values of velocity and density, numerical substitution indicates that the wave length should be long compared with the length of the ship. No attempt has been made by the authors to calculate the effect on the ship, but Captain A. Heinen, an experienced Zeppelin pilot, states that this condition is familiar to him and that he has never observed anything of violence about it. His practice is to proceed at full speed through windrow clouds.

It seems reasonable to conclude that gusts, turbulence and eddies near the ground need not be considered in the design of the airship except for the condition of handling on the ground and lying to the mooring mast. The problem is quite similar in its uncertainty to the selection of adequate ground tackle for mooring

a vessel in an open roadstead. A seaman knows when to cast off and put to sea to ride out the blow.

The existence of veritable fountains of upward rushing air whose sides at times and places are sharply separated from the surrounding atmosphere, must be taken account of in the design of the airship. The most violent of such currents, the tornado, combines vertical velocity with rotation, but fortunately can be seen from a great distance and can and must be avoided. The thunderstorm with large fully developed cumulus tops is also conspicuous and avoidable. It would appear to be folly to enter such a cloud and subject the ship to the unknown dangers of wind, rain, hail and lightning. Barring such spectacular and presumably avoidable hazards as these, there remain, at flying altitudes, convection currents with attendant turbulence which the ship may run into at full speed. There is ample evidence that upward velocities as high as three metres per second will be encountered. This vertical air velocity v, combined with the relative horizontal speed V of the airship, will give the effect of a change in pitch of $\tan^{-1} v/V$ or, say, 6 or 7 degrees. This is serious, but can be allowed for in the design by the methods of calculation previously explained.

The calculation for the ship flying through still air with a pitch angle to obtain dynamic lift has been previously described. The same calculation should apply for the case of such vertical air currents, but remembering that with a substantial trim by the stern the ship cannot make full speed, while now we consider a ship at full speed running suddenly into an up current of air. It is calculated that when the ZR.1 is proceeding at a speed of 86 ft./sec. and suddenly runs into a gust giving the effect of 6 degrees yaw or pitch, the maximum bending moment will be 318,000 metre-pounds, which is the greatest bending moment found in our calculations.

In the end the design of a ship, to be strong enough to withstand reasonably intelligent handling in reasonably unfavourable weather, must be based upon experience and comparison with many examples of successful and a few examples of unsuccessful practice. Ocean-going vessels are calculated in an arbitrary way with factors of safety (or experience) which do not involve the estimation of the true force of storm waves.

Forces on an Airship Lying to a Mooring Mast

When an airship is secured to a mooring mast, serious stresses may be produced by lack of attention to the static equilibrium. A condition of inequilibrium will not necessarily be indicated by the stern rising or falling; and the airship may, for example, have a large excess of weight over buoyancy combined with a condition of bow heaviness such that the excess of weight is balanced by a vertical reaction between the mast and the bow of the airship, producing serious sagging bending moments, but not inclining the axis of the airship from the horizontal. Such a state of affairs must be guarded against by careful control of the ballasting of the airship while lying to the mooring mast. The calculation of the shearing forces and bending moments, due to some assumed worst condition of ballasting when at the mast, presents no difficulties or peculiarities.

The most interesting and complicated problem in the investigation of mooring mast conditions is the calculation of the bending moment resulting from aerodynamic forces in the horizontal plane.

It is generally supposed that an airship lying to the mooring mast in a steady wind with rudders amidships will head directly into the wind, and that the only forces imposed will be longitudinal. In reality this is by no means the case. An airship lying head to the wind at the mast with rudders amidships is in a position of unstable equilibrium. If a slight angle of yaw occurs, a yawing moment is

produced, increasing the angle of yaw and still further increasing the yawing moment. For the average modern airship the maximum yawing moment occurs at about 3 degrees yaw and then reduces rapidly to zero at about 5 degrees. A further angle of yaw produces a moment tending to reduce the angle, so that this position of zero yawing moment is one of stable equilibrium.

Experience with the British rigid airships R.33 and R.36 at the mooring mast showed that there was a tendency for the airship to oscillate between the two positions of stable equilibrium, about 5 degrees either side of the wind direction. This oscillation might be fairly rapid in some circumstances, and the frequent reversal of the direction of the transverse forces and of the stresses appeared to be very undesirable. It was overcome by setting the rudders to an angle of about 5 degrees, giving the airship a definite angle of yaw to one side of the wind direction.

The angle at which an airship will lie to a steady wind at the mooring mast with the rudders set to any given angle may be determined from wind tunnel data. In such data, moments are usually given about the centre of buoyancy, but by transferring to moments about the bow, the angle of yaw corresponding to the equilibrium position of no yawing moment is readily found.

Having found the angle at which the airship lies to the mast, the distribution of transverse force may be calculated by Dr. Munk's formula for fixed angle of yaw. The concentrated forces at the bow and at the centre of pressure of the tail surfaces must be equal to each other and give equilibrium of moments with the forces along the bare hull. The resulting shearing forces and bending moments may be calculated in the usual manner.

When the wind direction changes, as in gusts, the angle of attack is momentarily altered. The new transverse air forces for any assumed change in the direction of the wind may again be calculated by Dr. Munk's formula; and equilibrium with additional air forces is given by increased transverse force at the bow and the inertia of the airship against the angular acceleration. The inertia forces may be calculated frame by frame, and they are proportional to the weight of the frame and the square of the distance from the bow. The magnitudes of the additional force at the bow and the inertia forces are such as will give equilibrium by moments and transverse force with the additional aerodynamic forces resulting from the gust.

It has been calculated from wind tunnel data that the position of equilibrium for the ZR.1 with rudders at 5 degrees, and neglecting the effect of the cars, is 7.9 degrees off the wind. At this angle in a wind of 88 ft./sec. velocity, the maximum mending moment was calculated by Munk's formula and found to be 130,000 metre-pounds.

Conclusions as to Aerodynamic Forces

It is concluded provisionally, until verified by full-scale tests now planned, that we have ground to assume that:—

(*a*) Munk's formula for distribution of transverse force on the hull at fixed angles of pitch may be used in lieu of wind tunnel pressure plotting data.

(*b*) Since the agreement under (*a*) is good, we are justified in using Munk's calculation for the distribution of force along the hull of an airship when turning. A wind tunnel determination is not practicable for this case, but comparison with turning trials of British airships shows that the radius of turn computed from Munk's formula is in very fair agreement with the radius estimated in the full-scale tests.

(*c*) German trials for dynamic lift are in fair agreement with the dynamic lift calculated either from wind tunnel pressure plottings or by Munk's formula.

(*d*) Munk's calculation for distribution of force can, therefore, be safely used in the direct calculation of the aerodynamic forces on the ship under *all* the various conditions of flight here discussed.

The maximum bending moments calculated for ZR.1, under the various conditions of static and aerodynamic loading, are summarised in the following table:—

Table of Maximum Bending Moments in the ZR.1 in Various Static and Dynamic Conditions

Static bending moments, metre-pounds:—	
Full load, best condition	97,500
Full load, unfavourable distribution	196,300
Light load	187,000
With one deflated gas cell	277,000
Dynamic bending moments:—	
6° pitch at 68 ft./sec. velocity	190,000
12° pitch at 48.5 ft./sec. velocity	190,000
6° pitch at 86 ft./sec. velocity (running into a gust) ...	318,000
Sudden application of the rudders at 20° at 86 ft./sec. velocity	107,000
Steady turning, 2,114 ft. radius at 85 ft./sec. velocity ...	230,000
Lying to mooring mast in steady wind of 88 ft./sec. velocity	130,000

All the dynamic conditions, except the case of sudden application of the rudders, are calculated by Munk's formulæ, and with the exception of the mooring mast condition, the altitude is assumed to be 6,000 feet.

Errors of not more than 20 per cent. may be expected in the calculations of the dynamic bending moments under the conditions assumed. It cannot be pretended that these conditions represent all the loadings that the ship may encounter, but they are typical of the various kinds of loading. The practical factor of safety required in design to allow for unfavourable combinations must come from experience. This experience cannot be analysed to give practical factors of safety until the structure of successful ships is calculated by a standard method such as this paper proposes.

SECTION IV.

Longitudinal Strength

The calculation of the stresses resulting from given primary shearing forces and bending moments has been the subject of more literature and discussion than any other problem connected with the strength of rigid airships. Exact computations of these stresses are very desirable, but owing to the extraordinarily high degree of redundancy of the hull structure, such computations must be based on the principle of least work, or on some equivalent method of deflections involving the elastic properties of the entire structure. The application of such methods to the hull as a whole has been found impracticable owing to the number of unknown terms and equations involved. Even the exact calculation for the simple case of a hexagonal braced structure, four frame spaces in length, and with symmetrical loading, requires the solution of ten simultaneous equations. Furthermore, the solution involves differences between quantities very nearly equal, so that unless the numerical work is correct to six or seven significant figures, very considerable errors are inevitable in the results. An exact computation by a similar method of the stresses in an airship structure would be vastly

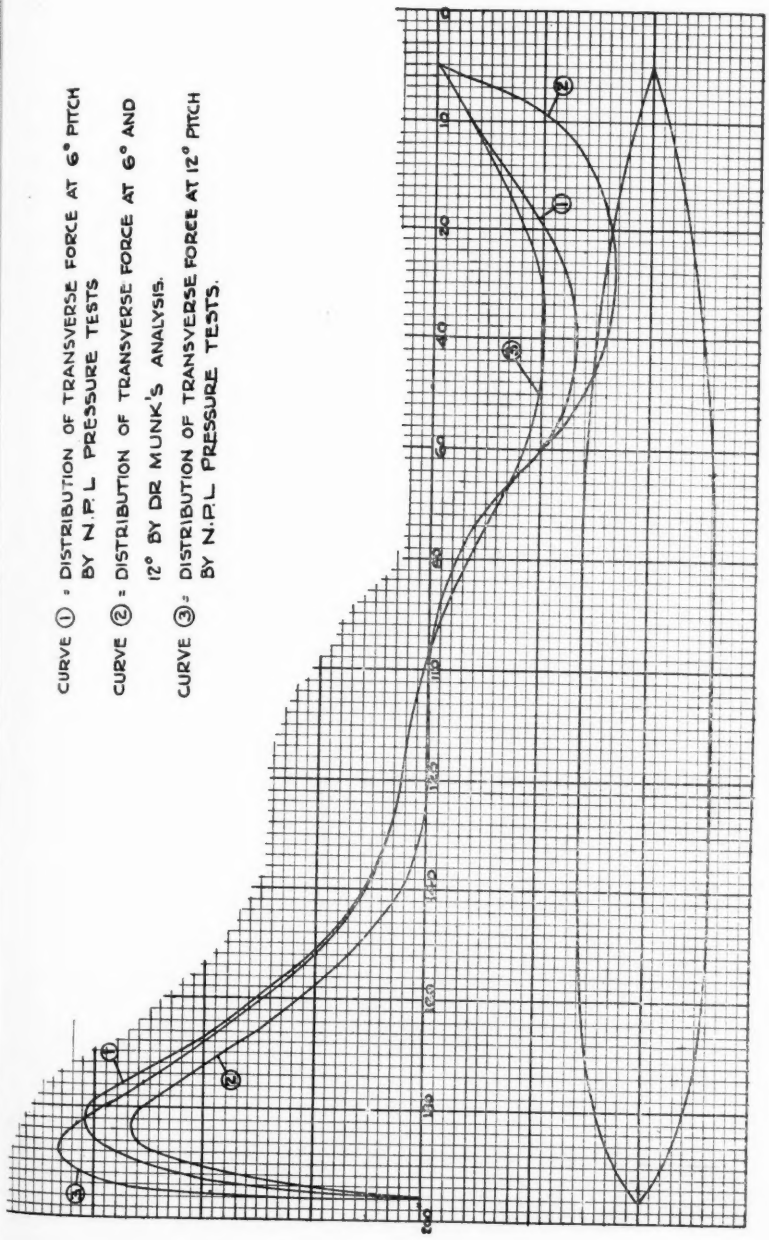


Fig. 17.
Distribution of transverse air forces at fixed angles of yaw or pitch, according to N.P.L. normal pressure experiments and Dr. Munk's analysis.

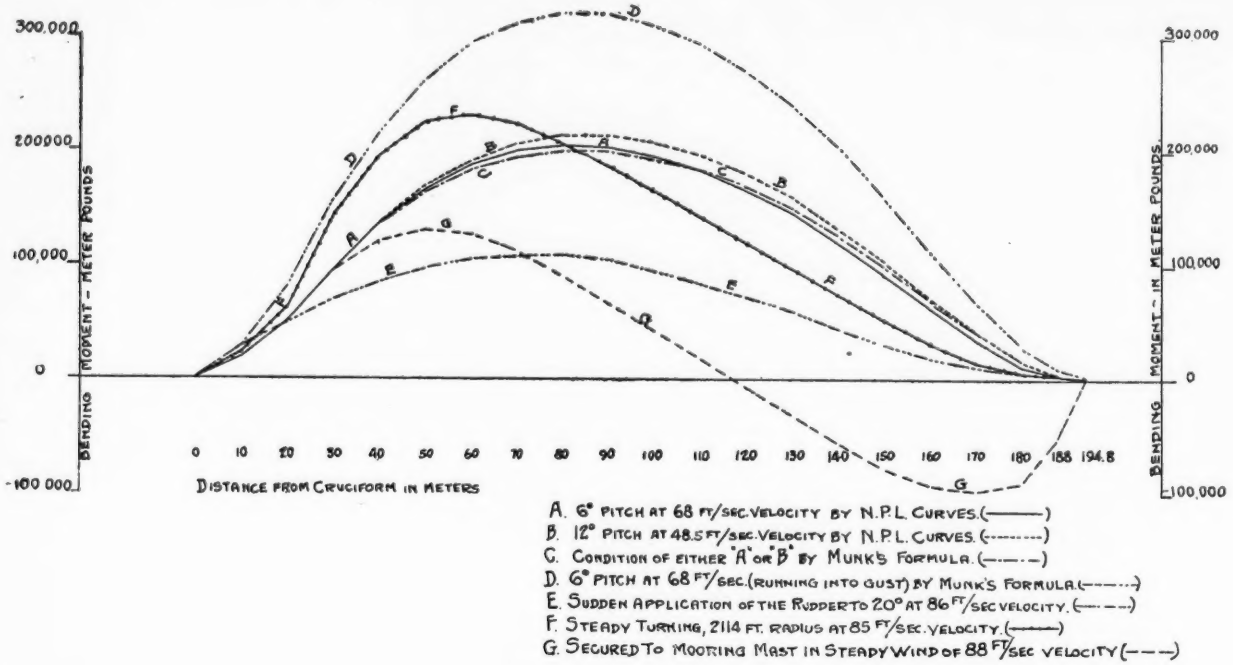

300,000
200,000
100,000
0
-100 000
BENDING MOMENT - METER POUNDS
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 188 194.8
DISTANCE FROM CRUCIFORM IN METERS
300,000
200,000
100,000
0
100,000
BENDING MOMENT - IN METER POUNDS
A. 6° PITCH AT 68 FT/SEC. VELOCITY BY N.P.L. CURVES. (———)
B. 12° PITCH AT 48.5 FT/SEC. VELOCITY BY N.P.L. CURVES. (--------)
C. CONDITION OF EITHER "A" OR "B" BY MUNK'S FORMULA. (—·—·—)
D. 6° PITCH AT 68 FT/SEC. (RUNNING INTO GUST) BY MUNK'S FORMULA. (———·)
E. SUDDEN APPLICATION OF THE RUDDER TO 20° AT 86 FT/SEC VELOCITY. (— — —)
F. STEADY TURNING, 2114 FT. RADIUS AT 85 FT/SEC. VELOCITY. (———)
G. SECURED TO MOORING MAST IN STEADY WIND OF 88 FT/SEC VELOCITY (— — —)

FIG. 18.

Curves of dynamic bending moment for airship ZR.1.

more difficult than in the comparatively simple case of the hexagonal braced tube, even if the modifications introduced by the rigidity of the joints are neglected.

Since exact computation appears to be impracticable, an approximate solution and a measure of the degree of precision of the approximation must be sought. Two approximate methods are well known to airship designers and various refinements have been suggested by different investigators, but without means for estimating the precision of the results.

The two approximations may be described as the methods of bending and of shear. It is found that the two methods give discordant results unless the sizes of the longitudinal girders and the shear wires bear a particular relation to each other. One of the special cases in which the two methods give concordant results is when the longitudinal girders are at the apeces of regular polygons, and the cross-sectional areas of the girders and the wires are uniform throughout each frame space. In actual airships considerable discrepancies are found between the results of the two systems of computation, especially in the lower part of the hull.

As will be shown, there is reason to believe that the true stresses lie somewhere between the results of calculations by modified bending and shearing methods, so that by calculating the stresses by both methods, a limit to the possible error is obtained.

In early attempts to solve the problem of longitudinal strength, it was usually assumed that the longitudinals took all the bending forces and the shear wires took all the shearing forces. In the bending method, which was probably suggested by analogy with the practice of naval architecture, the stresses in the longitudinals at each frame are calculated from the ordinary bending formula, $M/I=f/y$, the cross sectional area of the girders being used in the calculation of the moments of inertia of the cross sections of the hull. The structure of the hull is such that longitudinal forces are applied to the longitudinals only at the joints; and it is assumed that the tensions in the shear wires are such as to apply to the girders at each frame the change of force required by the bending theory. This procedure is equivalent to equating the longitudinal components of the tensions in the wires to the longitudinal shear in each panel, without regard to the sizes of the wires.

The following formula is easily derived for the wire tensions in the parallel middle body of the airship, assuming only one wire to be acting in each panel:—

$$T=Sb\Sigma Ay/2I\cos\phi,$$

where T = tension in a shear wire.

S = the transverse shear, assumed to be uniform throughout each frame space.
b = distance between frames.
A = cross sectional area of any longitudinal girder.
y = distance of any longitudinal from the neutral axis.
I = moment of inertia of the cross section of the hull.
ϕ = inclination of the shear wire to the longitudinal girders.

The summation ΣAy is taken over all longitudinal girders farther from the neutral axis than the wire for which T is desired.

The method of shears is based upon the elastic properties of the wires, and neglects these properties in the girders. It is assumed that the transverse shearing forces produce parallel vertical movements between consecutive frames without rotation or distortion of the frames. The tensions produced in the wires by the parallel movements depend on the elastic properties of the wires and their inclinations to the direction of the shearing force. By equating the total trans-

FIG. 18. *Curves of dynamic bending moment for airship ZR.1.*

verse shear to the sum of the components of the wire tensions in the direction of the shear, the following expression is obtained for the tensions in the wires:—

$$T = Sa \cos\theta / \sin\phi \Sigma a \cos^2\theta,$$

where a = cross sectional area of a shear wire.

θ = inclination of the panel containing a shear wire to the plane of bending.

The other symbols have the same significance as in the previous equation; and the summation, $\Sigma a \cos^2\theta$, is to be taken over all shear wires and panels in the frame space.

In calculating the stresses in the longitudinals by the method of shears, it is necessary to go through the laborious procedure of summing the forces applied by the shear wires to the longitudinals at the joints, working from end to end of the airship. No regard is given to the sizes or elastic properties of the girders.

Both the shearing and bending methods have had their advocates. Colonel Crocco, of the Italian Central Aeronautic Institute, who has made the only known water model experiments for rigid airships, adopted the bending method. Mr. E. J. Temple, formerly of Vickers, Ltd., favoured the shearing method. The late Mr. C. I. R. Campbell, R.C.N.C., adopted the compromise procedure of applying the bending theory for the stresses in the longitudinals, and the shear theory for the stresses in the shear wires. It is believed that Mr. E. H. Lewitt published the first discussion of both these methods of calculating the longitudinal strength of rigid airships. His articles on the subject appeared in the British aviation magazine, "Aeronautics," during 1919 and 1920.

The U.S. Navy Department retained Professor Wm. Hovgaard, of the Massachusetts Institute of Technology, in an attempt to place the theory of the longitudinal strength of rigid airships on a firmer basis than was evinced by the discordant theories of shear and bending. Professor Hovgaard declared himself in favour of the bending theory; but he pointed out that there was a fundamental error in the assumption that the longitudinals take all the bending forces; and he showed that the lengthening or shortening of the panels, due to the bending of the hull, throws forces into the shear wires, and the longitudinal components of these forces contribute to the strength of the hull against bending. Professor Hovgaard suggested that the assistance given to the longitudinals by the shear wires be calculated by assuming fictitious bars coincident with the longitudinals, and of the same material, and to be included in the calculation of the moment of inertia of the cross section of the hull. He showed that the areas of the fictitious bars could be computed by the formula:—

$$A_f = qa \cos^3\phi,$$

where A_f = the cross sectional area of a fictitious bar.

q = the ratio of the modulus of elasticity of the material of the wires to that of the girders.

a = the cross sectional area of the wire or wires on one side of a joint.

ϕ = the inclination of the wires to the longitudinals.

Professor Hovgaard has published his views in a paper, "The Longitudinal Strength of Rigid Airships," read before the American Society of Naval Architects and Marine Engineers in November, 1922.

Mr. W. W. Pagon, who was a member of the Special Committee of the National Advisory Committee for Aeronautics appointed to review the design of the ZR.1, has proposed formulæ for the stresses in the longitudinals and the shear wires based on the assumption that as a result of the transverse shear and the bending moment, a combined parallel and angular movement occurs between consecutive transverse frames, without distortion of the frames. He takes account of the elastic properties of both the longitudinals and the shear wires, and calculates the forces produced in these members by the movement which will give

equilibrium between these forces and the combined transverse shear and bending moment. But unless the sizes of the longitudinals and shear wires bear a particular relation to each other, it can be shown that the calculated forces on either side of the joints are not in equilibrium, and the assumption that the transverses remain plane is therefore unsound.

Mr. R. V. Southwell has advocated that the stresses produced by the shearing forces and bending moments be calculated separately, and then added together in accordance with the principle of superposition. He considered the case of an open-ended braced cylindrical structure, with a uniform shearing force and a bending couple applied to one end of the structure by longitudinal forces distributed to the joints in accordance with the requirements of the bending theory. He showed that the stresses due to the bending moment would then be correctly given by the bending theory when including Hovgaard's fictitious bars, irrespective of the relation between the sizes of the longitudinals and wires. Exact analysis by deflections shows that Southwell's solution would not apply when the shear varies from one frame space to the next unless the sizes of the longitudinals and wires bear the particular relation to each other which is required for a correct result from the methods of shear and bending. Mr. Southwell's theory is set forth in A.R.C. Report, R. and M. 791.

It will now be shown that if allowance be made for the bending forces taken by the wires, the actual stresses in both the longitudinals and the wires will lie between the limits determined by the methods of shear and bending.

Since the forces in the longitudinals are due to the forces applied by the shear wires at the joints, it is possible to choose shear wires proportioned to any system of longitudinals in such a manner that the tensions in the wires will so load the longitudinals that the frames remain plane.

With plane frames the conditions are satisfied for the stresses in the longitudinals and in the wires due to the bending to be correctly given by the bending theory, and for the stresses in the wires due to the transverse shear to be correctly given by the shear theory. Since the forces at the joints must be in equilibrium, the stresses in the wires are also in accordance with the longitudinal shear as determined by the bending theory; and the stresses in the girders are in accordance with the forces applied by the wires to the joints as determined by the shear theory. It follows that when the sizes of girders and wires are so proportioned to each other that the frames remain plane, the stresses are correctly given by either the shear or bending theory.

Suppose now that a longitudinal be replaced by a new one of different cross sectional area without other change in the structure. The new longitudinal will yield either more or less than the old one, under the forces applied by the shear wires, according to whether it is smaller or larger than the old one. If it is larger and yields less, the unit stress and total force will be less than by the bending theory; and conversely if it is smaller. At the same time the smaller yielding of the larger longitudinal will cause an increase of the forces applied to it by the shear wires, so that the total force in the longitudinal will be greater than by the shear theory.

It is thus evident that if the new longitudinal is larger than the old one without other change in the structure, the force in it will be less than by the bending theory and more than by the shear theory, and conversely if it is smaller.

By a similar argument, it may be shown that the tensions in the wires lie between the limits determined by the shear and bending theories. No definite rule is known for determining which limit the stress will be nearer to; but, in general, large bending and shearing forces are not found in the same frame space, except in the case of a deflated gas cell; and where the bending moment is relatively more important it may be expected that the stresses in the longitudinals will be more nearly in accordance with the bending than the shear

theory. Where the shearing forces are relatively more important it may be expected that the tensions in the shear wires will approximate more closely to the shear theory. As we are most concerned with the girder stresses where the bending is large, and with the wire stresses where the shear is large, the late Mr. Campbell's practice of calculating the girder stresses by the bending theory and the wire stresses by the shear theory has much to recommend it, although it is likely to lead to an over estimate of the maximum stresses.

A good practical method for calculating the stresses in the longitudinals is believed to be the application of the bending theory with correction factors based upon the magnitude of the discrepancy between the shear and bending theories. An advantage of using correction factors is that they will almost certainly be found very useful, or even necessary, when full-scale experiments shall have furnished data on the departures from the bending theory which occur in actual practice. As an example, suppose that owing to the tapering off of the sizes of the shear wires in the upper part of the hull, the longitudinal shear which the wires in the top panels are capable of taking, assuming their tensions to be in accordance with the transverse shear theory, is only 80 per cent. as large as is required by the bending theory. The true stresses in the top longitudinal will, therefore, be somewhere between 80 per cent. and 100 per cent. as large as by the bending theory. Since we are most concerned with the case where the bending moment is large relatively to the shearing force, the correction factor should be taken as nearer to 1.0 than 0.8, and 0.93 appears to be a number which will reduce the probable error within a limit not greater than 3 per cent. either way. In the calculation of the longitudinal strength of the airship, the effective area of the top longitudinal will, therefore, be taken as .93 times the actual area, and the bending theory will be applied on this basis.

The extension of the correction factor method to all the longitudinals of the ZR.1 is described in Appendix IV., and the calculated stresses with and without these factors are compared.

Section V.

Secondary Stresses Resulting from the Primary Shearing Forces.

The longitudinal girders are continuous over transverse frames to which they are fastened by rigid joints and, therefore, must bend slightly to follow the contour of the axis of the ship when the latter bends as a whole. Under the theory of bending we assume that the main frames remain plane, but rotate under the action of the primary bending moment. The additional or secondary stress thrown into the longitudinal girders, through the rigid joints, by rotation of the main frames, could be allowed for in the calculation of the primary stresses if we used the distances of each channel element of a girder from the neutral axis of the ship, instead of taking an average distance for the girder. However, since the depth of any girder near the top or bottom of the ship is small with respect to its distance from the neutral axis, and since we are interested here only in these highly stressed girders, the refinement obtained by allowing for this secondary stress is of no importance and is neglected.

The secondary stresses due to the relative movements of the main transverses under the primary shearing forces remain to be considered. Mr. W. W. Pagon has given a very complete treatment of the secondary stresses in connection with the review of the design of the ZR.1 by the special committee of the National Advisory Committee for Aeronautics. He assumed that the transverse girders offer no resistance to torsion, but that they resist rotation of the joints by a kind of portal action. In order to determine the secondary stresses exactly, it would be necessary to consider the shearing movements of the transverses over the entire length of the hull, and the calculations would be very laborious. Mr.

Pagon obtains an approximate solution by considering that the longitudinals are normal to the main frames next forward and aft of the one at which the secondary stresses are desired. His treatment of the problem is set forth in a pamphlet of the National Advisory Committee for Aeronautics.

The authors propose to divide the secondary bending of the longitudinals into two parts, one in a radial plane of the hull, including the longitudinal axis of the ship and a longitudinal girder, and the other in a longitudinal plane, designated the tangential plane, including a longitudinal girder and normal to the radial plane. At the top and bottom of the hull (when the primary bending is in the vertical plane) the secondary bending is wholly radial, and is wholly tangential only at the mid-height of the hull. Radial secondary bending occurs, therefore, where the primary stresses are greatest and tangential bending where they are least. Moreover, the depth of the longitudinals is much greater than the breadth, so that the maximum secondary stresses in the radial plane are much larger than the maximum in the tangential plane. For these reasons the secondary stresses in the tangential plane may be neglected.

Since the transverse girders offer practically no resistance to torsion, the longitudinals may be considered as continuous in the radial plane over the joints, but not fixed. The secondary stresses will then be the same on both sides of a joint. Assuming the longitudinals to be normal to the main transverses next forward and aft of the one at which the secondary stresses are desired, it follows from the well-known relations between stress and deflection in a fixed ended beam with a central load and the ends at different levels, that the secondary stress is given by

$$s = 3\,Ey\,(\Delta_1 + \Delta_2)/b^2,$$

where s = secondary stress in a channel of a girder.
y = the distance of the channel from the neutral axis of the girder.
b = distance between main frames.
Δ_1 and Δ_2 = the radial movements of the girder at the main frames next forward and aft, respectively, relative to the frame at which s is taken.

From the transverse shear theory of the primary stresses, the relative movement between consecutive main frames is given by

$$d = Sb/E\Sigma a \,.\, \sin^2\phi \,.\, \cos\phi \,.\, \cos^2\theta,$$

where d = the relative movement, and the other symbols have the same significance as in the transverse shear theory.

It follows that

$$\Delta_1 + \Delta_2 = (S_1 - S_2)\, b \cos u/E\Sigma a \,.\, \sin^2\phi \,.\, \cos\phi \,.\, \cos^2\theta,$$

where S_1 and S_2 are the transverse shearing forces forward and aft, respectively, of the frame under consideration, and u is the angle between the vertical and the radial plane through the longitudinal girder.

The secondary stress is given by

$$s = 3y\,(S_1 - S_2)\cos u/qb\Sigma a \,.\, \sin^2\phi \,.\, \cos\phi \,.\, \cos^2\theta,$$

where q is the ratio of the modulus of elasticity of steel to that of duralumin.

As an example of the maximum secondary stress due to shear likely to occur in the ZR.1, the case of full load with a deflated gas cell between frames 80 and 90 may be taken. In this condition the transverse shear changes at frame 90 from 6,754 lbs. to −6,457 lbs. The given quantities are then

$$S_1 = -6{,}457 \text{ lbs.}$$
$$S_2 = 6{,}754 \text{ lbs.}$$
$$q = 2.86$$
$$b = 394 \text{ inches}$$
$$\Sigma a \,.\, \sin^2\phi \,.\, \cos\phi \,.\, \cos^2\theta = .0571 \text{ in.}^2$$

And for the A longitudinal

$$y = 8.14 \text{ in. (apex channel)}$$
$$\cos u = 1.0$$

The secondary stress in the apex channel of the A longitudinal is then given by

$$s = 3 \times 8.14\,(-6457 - 6754)/2.86 \times 394 \times .0571$$
$$= 5{,}000 \text{ lbs./in.}^2$$

The important secondary stresses in the longitudinals due to the rise and distortion of the intermediate transverses are not here considered because it is more convenient to defer analysis of these stresses to the section of this paper on the effect of gas pressure. This effect is local rather than connected with the general deflection of the ship as a whole.

Section VI.

Effect of Gas Pressure

The local transverse loads imposed on the longitudinals by the gas pressure produce stresses of approximately the same order of magnitude as the maximum resulting from the primary bending moment in the hull, and satisfactory methods for computing them are of great importance. The authors have developed theories and methods of procedure and have applied them in the strength calculations of the ZR.1. The results of calculation have been compared with measurements made in a series of experiments in which a gas cell in the parallel middle body of the actual airship was inflated with helium. In these experiments over 13,000 readings were taken, including strains in longitudinal and transverse girders, tensions in diagonal and transverse wires, movements of the joints of main and intermediate frames, and curvature of the wire netting between longitudinals. While a complete description and analysis of the experiments would be far too voluminous for this paper, it is proposed to give such figures and analyses as are necessary for a fair comparison between the calculated and observed loads, deflections and stresses.

As a preliminary to a discussion of the experimental results, the theory of the subject will be described.

Theoretical Views

The fabric of the gas cell is forced by the gas pressure against the bases of the longitudinal girders and against the netting which is secured to the girders. It is sometimes thought that the pressure of the gas on each panel must be taken by the longitudinals as radial loads; but in reality this would be true only if the netting bulged in a complete semi-circle (assuming that the shear wires and outer cover permitted this shape) as shown in Fig. 19. In this case the netting would impose no loads upon the longitudinals tangentially to the cross-section of the hull.

Another special case is when the netting has the same curvature as the cross-section of the hull, also shown in Fig. 19. The netting is everywhere tangential to the hull, so that the tension has no component throwing radial loads upon the girders. There are, however, tangential loads upon the longitudinals resulting from the increase of the gas pressures and tensions in the nettings upward from panel to panel.

The manner in which the lift of the gas is transmitted to the hull differs very greatly in these two cases. In the first, the lift is due to the radial outward loads upon the upper longitudinals; while in the second case, the lift is the result of the tangential forces, mainly on the longitudinals at the mid-height of the hull.

The effect of the variation of curvature of the netting is of great importance; and a similar problem will occur when considering the loads imposed on the longitudinals by the outer cover, resulting from differences between the external and internal air pressures. The point is worth some emphasis, because it is often argued that the curvature of the netting can make no difference in the loads on the longitudinals, just as the curvature of the cables of a suspension bridge makes no difference in the loads upon the towers. The fallacy in this argument may be explained by reference to Fig. 20. The radial load each panel of netting puts upon the longitudinal to which it is attached is given by

$$w = t \cos \psi,$$

where w = the radial load which the netting puts upon the longitudinal per unit length.

t = the tension in the netting per unit width.

ψ = the angle between the tangent to the netting and the radial plane through the longitudinal.

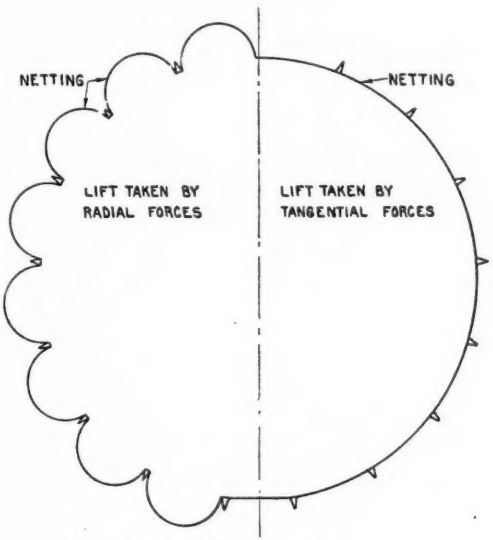


FIG. 19.

Illustrating effect of curvature of netting on distribution of lift forces.

And from the figure

$$\psi = \pi - \gamma - \lambda$$

and

$$t = pr,$$

where p = gas pressure per unit area, assumed constant for the panel.

r = transverse radius of curvature of the netting.

Therefore,

$$w = pr \cos (\pi - \gamma - \lambda).$$

In the special case when $\lambda = \pi/2$,

$$w = pr \cos (\pi/2 - \gamma),$$
$$= pr \sin \gamma,$$

and $\sin \gamma = c/2r$, where c = the width of the panel. Therefore

$$w = pc/2.$$

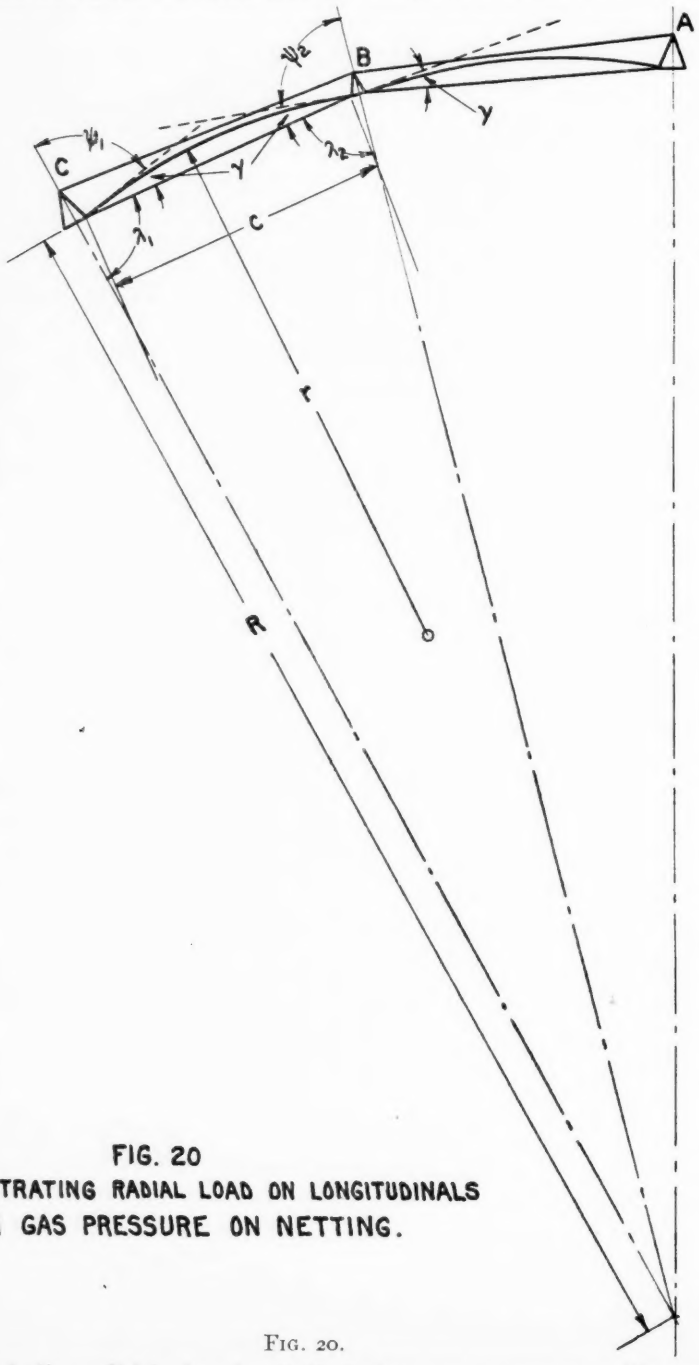


FIG. 20
ILLUSTRATING RADIAL LOAD ON LONGITUDINALS
FROM GAS PRESSURE ON NETTING.

FIG. 20.

Illustrating radial load on longitudinals from gas pressure on netting.

This case applies to the load upon the towers of a suspension bridge, but is obviously not generally applicable to the radial loads upon the longitudinals of an airship.

Another special case is when $r=R$, then $\gamma+\lambda=\pi/2=\psi$; therefore $w=0$.

In airships, such as the ZR.1, where the main and secondary longitudinals are of different depths, λ is larger at the main than at the secondary longitudinals, and consequently the radially outward loads are greater on the main longitudinals.

The curvature of the netting is usually such as to transmit the lift of the gas to the hull by a combination of radial and tangential loads upon the longitudinals. It is possible to brace the longitudinals much more efficiently in the tangential than in the radial direction; and, in fact, the netting, when composed of diagonal wires, is in itself a tangential bracing. It appears, therefore, that the netting should be strong and taut enough to bulge but little beyond the tangents to the longitudinals, so that the radial loads will be small. In addition to the loads from the netting, there are small radial loads due to the direct gas pressure upon the bases of the longitudinals.

To determine the loads from the netting it is necessary to calculate the tensions in the netting and the angle γ for each panel. These quantities depend on the curvature of the netting, and this in turn depends on the strength of the wiring and the manner in which it is initially set up.

Tension in the Wire Netting

If a uniformly distributed transverse load is applied to a wire initially taut, between immovable supports, the resulting tension in the wire is given by

$$24\,T^3-24\,T_0T^2=EaP^2,$$

where T_0 = the initial tension,
T = the final tension.
E = modulus of elasticity.
a = cross-sectional area.
P = total transverse load.

This expression is derived in Technical Note No. 192, "Tension in a Wire Loaded Transversely," of the Bureau of Construction and Repair, Navy Department.

If the wire be initially slack, T_0 is negative, although there cannot actually be compression in the wire. T_0 may then be considered as a virtual compression connected with the slack in the wire by the relation

$$T_0=EadL/L,$$

where L is the distance between the terminals and dL is the extra length of wire which gives the slack.

In ZR.1 the netting wires are set up as nearly as possible just taut, so that $T_0=0$ and $24T^3=EaP^2$.

In calculating P, it may be assumed without appreciable error that the unit gas pressure is uniform over each panel. P equals the unit pressure multiplied by the length of the wire and the distance between wires, and multiplied again by .5 if there are wires running in two directions, as in a netting.

In Appendix V. it is shown that in the parallel middle body of the ZR.1, $P=14.15\,p$, $T=17.2\,P^{2/3}=100.6\,p^{2/3}$, and $t=.4875\,T=49\,p^{2/3}$.

Since the deflection, e, of the netting is small compared with c, $r=c^2/8\,e$; and $t=pr=pc^2/8\,e$. Whence $e=2.32\,p^{1/3}$, where e is expressed in inches and p in lbs./ft.^{2}.

In the gas cell test of the ZR.1, the cell was filled with helium lifting .063 lbs./ft.^{3}. Table 1 shows the calculated and observed deflections of the

netting when the cell was fully inflated without super-pressure. The actual deflections varied considerably among the four panels observed at each height, due doubtlessly to small differences in the initial set up of the wires; but it will now be shown that in spite of these variations the actual loads upon the longitudinals did not depart very greatly from the calculated ones.

As typical examples, the radial loads upon the A and C longitudinals will be considered. For these members, $\lambda = 85.65°$, $\psi = 180° - 85.65° - \gamma = 94.35° - \gamma$, and $\gamma = \sin\gamma = c/2r = 4\,e/c = .0383\,e$ radians $= 2.19\,e$ degrees. The magnitudes of γ, ψ, $\cos\psi$, t and w, based on calculated and observed values of e in panels AB, BC and CD are shown in Table 2. The corresponding total radial loads, including the direct pressure on the base, for longitudinals A and C are shown in Table 3. The percentage departures of the observed from the calculated loads are also shown in this table.

In Appendix V. the calculations of the radial and tangential loads upon the longitudinals are described in detail, and a complete computation is given of the loads and stresses resulting from inflation with gas lifting .068 lbs./ft.^{3} with 2.1 lbs./ft.^{2} super-pressure.

Supporting Effect of Intermediate Transverses

There has been considerable divergence of opinion as to the amount of support which the longitudinals may be considered to derive from the intermediate transverses. Some have argued that the longitudinals should be regarded as fixed-ended at both the main and intermediate transverses; while others have held that the intermediate transverses give only a partial support tangentially and none radially. These may be regarded as the extreme views. The procedure of the authors is to calculate the stresses on the assumption that the longitudinals are fixed-ended at the main and intermediate frames, and to make a separate calculation for the effect of the movements of the joints of the intermediate frame. A theory of these movements and the movements actually observed in the gas cell test of the ZR.1 will now be considered.

The belief is very widely held, even among engineers familiar with airships, that under the influence of the buoyancy of the gas, the cross-sections of any airship, whether rigid or non-rigid, tend to assume a pear shape, the inverse of a suspended bag filled with water. This is sometimes called the "natural shape" of an airship, and it has even been proposed to construct rigid airships of this shape, in the belief that the stresses due to gas pressure would be less than with circular cross-sections. The explanation of this so-called "natural shape" is that the product of the unit gas pressure and the radius of curvature of the cross-section should be constant, giving a constant transverse tension all around the cross-section. In reality this case occurs only at a section where the gross lift equals the load suspended at the bottom. When the lift is taken wholly by shear or loads uniformly distributed around the circumference of the section, a non-rigid envelope will assume a circular shape, because in this case the circumferential forces, due to the shear or loads, cause the transverse tension to vary in exactly the same manner as the gas pressure.

The mathematical theory of the shape of non-rigid airship envelopes is very fully discussed in Report No. 16 of the National Advisory Committee for Aeronautics, 1917, "The Stretching of the Fabric and the Deformation of the Envelope in Non-Rigid Balloons," translated from the German of Rudolf Haas and Alexander Dietzius.

In a rigid airship the weights in the keel and the cars are applied to the structure almost entirely through the main transverses; and elsewhere in the hull the lift of the gas is opposed by shear and by weights almost uniformly distributed around the cross-sections. It is, therefore, to be expected that under the

influence of gas pressure the intermediate frames will rise bodily with respect to the main frames, but without departing from the circular shape, except for a local distortion at the bottom where the upward movement is prevented by the keel. The authors have, therefore, adopted the procedure of calculating the rise of the intermediate transverse on the assumption that there is no change of shape. The stresses in the longitudinals, due to any given rise of the intermediate transverse frame, are calculated by the customary formulæ connecting the deflections and stresses in fixed-ended beams.

The calculation of the force required to lift the intermediate transverse 1in. is given in Appendix V. It is shown that this force is 29,340 lbs. When inflated with helium, lifting .063 lbs./ft.^{3}, the rise of this frame should be .126 inches.

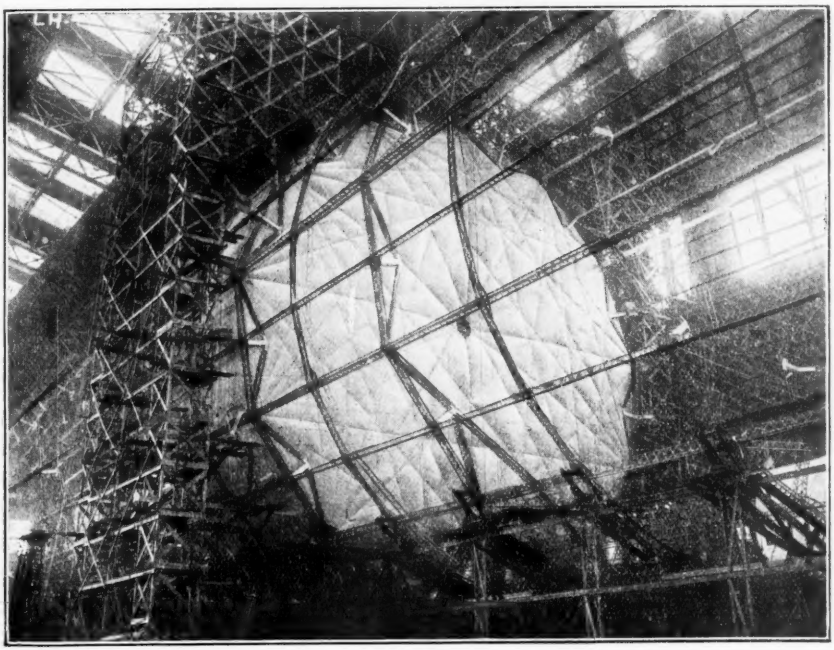


FIG. 21.

Gas cell inflation test on ZR.1.

The observed motions of the intermediate frame in the gas cell test of the ZR.1 are shown in Table 4. The movements were measured by means of pantographs secured to each joint, multiplying the motion eight times. The square boards carrying the sheets of paper on which the movements were recorded may be seen in Fig. 21 secured to the wooden scaffolding erected over the cell on which the tests were made.

Inspection of the figures of Table 4 shows that the vertical movements of the joints above the L and L′ longitudinals did not vary greatly. The average rise of these joints was .126 in., or precisely the same as calculated. The exceptional movements of the C and J joints are probably due to the changes in the sizes of the shear wires at these positions. The large radially outward movements of

the G and G′ joints are unaccounted for; but they are directly contrary to the widely held theory that the sides of the frame draw inwards.

Effect of Gas Pressure on Shear Wires

The tensions in the main and secondary shear wires between main frame §100 and the intermediate frame §105 are shown in Tables 5 and 6. The differences between the initial and final tensions are the stresses due to the inflation of the cell. The sum of the vertical components of these stresses is 813 lbs. From the calculation in Appendix V. of the resistance offered by the shear wires to the rise of the intermediate frame, the force required to lift the frame .126in. against the resistance of the wires on either the forward or aft side of the frame

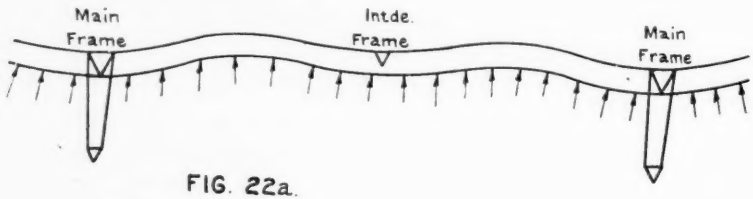


FIG. 22a.

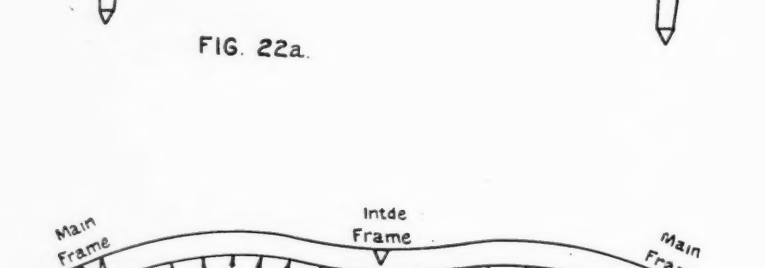


FIG 22 b.

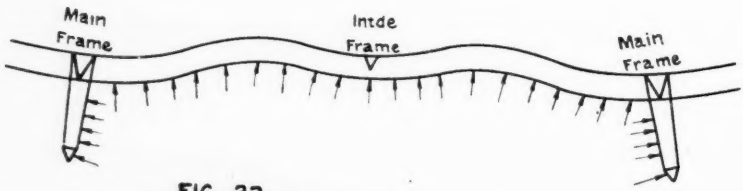


FIG. 22c

ILLUSTRATING BENDING OF LONGITUDINALS UNDER GAS PRESSURE LOADS.

is .126 × 7,000 = 882 lbs., which compares very well with the recorded 813 lbs. Although the agreement between theory and experiment is very good, as regards the total shearing force taken by the wires, it may be seen from the tables that the forces are very irregular when taken panel by panel.

It may seem surprising at first sight that the final tensions are on the average considerably less than the initial ones; but this is accounted for by the reduction in the diameter of the main frames under the heavy loads imposed upon them by the tensions in the transverse wires due to the pressure of the ends of the gas cell. The reduction in diameter averaged about .66 inches (see the section of this paper on Transverse Strength), and the authors have calculated that this would reduce the tensions in the main shear wires in panels AC, CE, KM, by 45 lbs., and the actual average reduction of tension was 32 lbs. In panels EG, GI and IK the calculated mean reduction is 56 lbs. and the actual average was 42 lbs. For the secondary shear wires the calculated reduction is 63 lbs. compared with 46 lbs. observed.

Effect of Gas Pressure on Longitudinals

In the gas cell test the end conditions for the longitudinals differed considerably from those which would occur if the neighbouring cells were inflated. In the latter case the symmetry of the gas pressure forces forward and aft of each main transverse makes the longitudinals virtually fixed-ended at the joints, as shown in Fig. 22a. It was expected that in the gas cell test the absence of gas pressure in the neighbouring cells would cause the longitudinals to bend somewhat as indicated in Fig. 22b. Actually it was found that they took the form represented in Fig. 22c. This surprising result was due to the pressure of the ends of the cell on the kingpost of the main frame, causing a rotation of the joint as indicated in the figure.

The calculation in Appendix V. shows that the longitudinals contribute only 11.7 per cent. to the total resistance of the structure to the rise of the intermediate transverse, so that the difference between the forces in the longitudinals in the conditions of the test, and with all cells inflated, should make but little difference in the movements of the frame.

TABLE 1.

COMPARISON OF CALCULATED AND OBSERVED DEFLECTIONS OF GAS CELL NETTING.

	Height	Unit Pressure		Calculated deflection =	Observed deflections. Starboard.		Port.	
	H	p		$2.32 \times p^{1/3}$	100-105	105-110.	100-105.	105-110
Panel.	ft.	lbs./ft.^{2}.	$p^{1/3}$.	in.	in.	in.	in.	in.
AB	77.5	4.88	1.70	3.94	3.44	5.25	4.62	4.87
BC	75.0	4.73	1.68	3.90	3.00	3.12	4.19	3.62
CD	70.5	4.45	1.64	3.80	4.19	4.00	3.00	1.87
DE	63.8	4.02	1.59	3.69	5.37	5.00	5.12	5.12
EF	55.7	3.52	1.52	3.52	4.44	4.37	2.12	3.75
FG	46.7	2.95	1.43	3.32	3.53	3.50	3.00	3.25
GH	37.0	2.33	1.33	3.08	3.25	2.75	3.12	2.62
HI	27.6	1.74	1.20	2.78	2.94	2.69	2.31	2.37
IJ	18.8	1.18	1.06	2.46	2.00	1.69	2.00	2.62
JK	11.15	0.70	0.89	2.06	2.00	2.06	2.44	2.69
KL	5.31	0.33	0.69	1.60	0	0.06	0.34	0.44
LM	1.57	0.10	0.46	1.07	0	0	0	0

TABLE 2.

COMPARISON OF CALCULATED AND OBSERVED TENSIONS AND LOADS IN PANELS AB, BC AND CD.

Panel.	AB				BC				CD			
p lbs./ft$_2$	4.88				4.73				4.45			
e (calculated) in.	3.04				3.00				3.80			
e (observed) in.	3.44	5.25	4.62	4.87	3.00	3.12	4.19	3.62	4.19	4.00	3.00	1.87
γ (calc.) deg.	8.04				8.56				8.35			
γ (obs.) deg.	7.55	11.50	10.13	10.70	6.59	6.85	9.20	7.95	9.20	8.78	6.58	4.10
Ψ (calc.) deg.	85.71				85.79				86.00			
Ψ (obs.) deg.	86.80	82.85	84.22	85.05	87.76	87.50	85.15	86.40	85.15	85.57	87.77	90.25
cos Ψ (calc.)	.0747				.0735				.0698			
cos Ψ (obs.)	.0558	.1245	.1008	.1106	.0390	.0436	.0846	.0628	.0846	.0773	.0390	−.0044
t (calc.) lbs.	141				138				133			
t (obs.) lbs.	161	106	120	114	179	172	128	149	121	127	169	270
w (calc.) lbs.	10.52				10.15				9.29			
w (obs.) lbs.	9.00	13.2	12.1	12.6	6.98	7.50	10.82	9.36	10.25	9.81	6.58	1.19

TABLE 3.

COMPARISON OF CALCULATED AND OBSERVED RADIAL LOADS ON LONGITUDINALS A AND C.

Longitudinal.	A	C
Pressure, p lbs./ft.^{2}	4.90	4.61
Direct radial load, lbs./ft.	5.10	4.80
Calculated radial load due to netting, lbs./ft.	21.04	19.44
Total calculated radial load, lbs./ft.	26.14	24.24
Total radial load from observed values of e, lbs./ft. ...	23.1	22.03
,, ,, ,, ,, ...	31.5	22.11
,, ,, ,, ,, ...	29.3	22.20
,, ,, ,, ,, ...	30.3	12.97
Average radial load from observed values of e, lbs./ft.	28.55	22.11*
Average departure from calculated radial load, per cent.	9.23	−8.78*
Maximum positive departure, per cent.	20.5	—
Maximum negative departure, per cent.	−11.6	−9.10*

* Exceptional departure on C longitudinal due to an excessively small recorded deflection in CD panel is omitted.

TABLE 4.

MOVEMENTS OF JOINTS OF INTERMEDIATE TRANSVERSE FRAME OF ZR.1 WITH GAS CELL FULLY INFLATED.

	Port Side.				Starboard Side.				
Joint.	Radial motion. in.	Tangential motion. in.	Vertical motion. in.	Horizontal motion. in.	Joint.	Radial motion. in.	Tangential motion. in.	Vertical motion. in.	Horizontal motion. in.
A	.151	—	.151	—	A	.151	—	.151	—
B′	.145	.020	.146	.0163	B	.110	.040	.116	−.012
C′	.230	.020	.212	.092	C	.210	.080	.223	.030
D′	.030	.045	.0525	−.013	D	.0	.095	.064	−.070
E′	.060	.043	.069	.030	E	.040	.085	.093	−.013
F′	.100	.075	.103	.071	F	.0	.100	.095	−.032
G′	.402	.128	.159	.390	G	.210	.075	.091	.203
H′	−.042	.110	.116	−.023	H	−.067	.092	.102	−.050
I′	−.200	.050	.127	−.163	I	−.195	.065	.139	−.152
J′	−.242	.070	.205	−.147	J	−.215	.075	.192	−.123
K′	−.142	.020	.125	−.071	K	−.015	.050	.042	.031
L′	−.132	−.040	.105	−.089	L	.075	.035	−.055	.062
M′	.090	.043	−.082	.057	M	.100	.025	−.095	.040
N	.038	—	−.038	—	N	.038	—	−.038	—

TABLE 5.

TENSIONS IN MAIN SHEAR WIRES AND VERTICAL COMPONENTS OF FORCES IN FRAME SPACE 100-105.

Wire.	Tensions. Initial. lbs.	Final. lbs.	Change. lbs.	$\sin\phi$	$\cos\theta$	Vertical component of change of tension. lbs.
A-100 to B-105 ...	140	85	−55	·5075	·1240	3·5
C-100 to B-105 ...	165	145	−20	·5075	·3645	−3·7
C-100 to D-105 ...	185	120	−75	·5075	·5825	22.2
E-100 to D-105 ...	160	165	5	·5075	·7646	1.9
E-100 to F-105 ...	240	125	−115	·5075	·8998	52.5
G-100 to F-105 ...	230	220	10	·5075	·9795	5.0
G-100 to H-105 ...	270	155	−115	·5075	·9990	58.4
I-100 to H-105 ...	240	240	0	·5075	·9569	.0
I-100 to J-105 ...	255	155	−100	·5075	·8560	43.5
K-100 to J-105 ...	220	220	0	·5075	·7025	.0
K-100 to L-105 ...	230	165	−65	·5075	·5056	16.7
M-100 to L-105 ...	310	290	−20	·5075	·2781	−2.8
					Sum.	197.2 lbs.
A-100 to B′-105	275	200	−75	·5075	·1240	4·7
C′-100 to B′-105	175	195	20	·5075	·3645	3·7
C′-100 to D′-105	185	160	−25	·5075	·5825	7·4
E′-100 to D′-105	175	220	45	·5075	·7646	17·5
E′-100 to F′-105	225	145	−80	·5075	·8998	36.6
G′-100 to F′-105	220	240	20	·5075	·9795	10.0
G′-100 to H′-105	290	190	−100	·5075	·9990	50.7
I′-100 to H′-105	290	260	−30	·5075	·9569	−14.6
I′-100 to J′-105	300	155	−145	·5075	·8560	63.0
K′-100 to J′-105	255	200	−55	·5075	·7025	−19.6
K′-100 to L′-105	200	200	0	·5075	·5056	.0
M′-100 to L′-105	260	250	−10	·5075	·2781	−1.4
					Sum.	158.0 lbs.

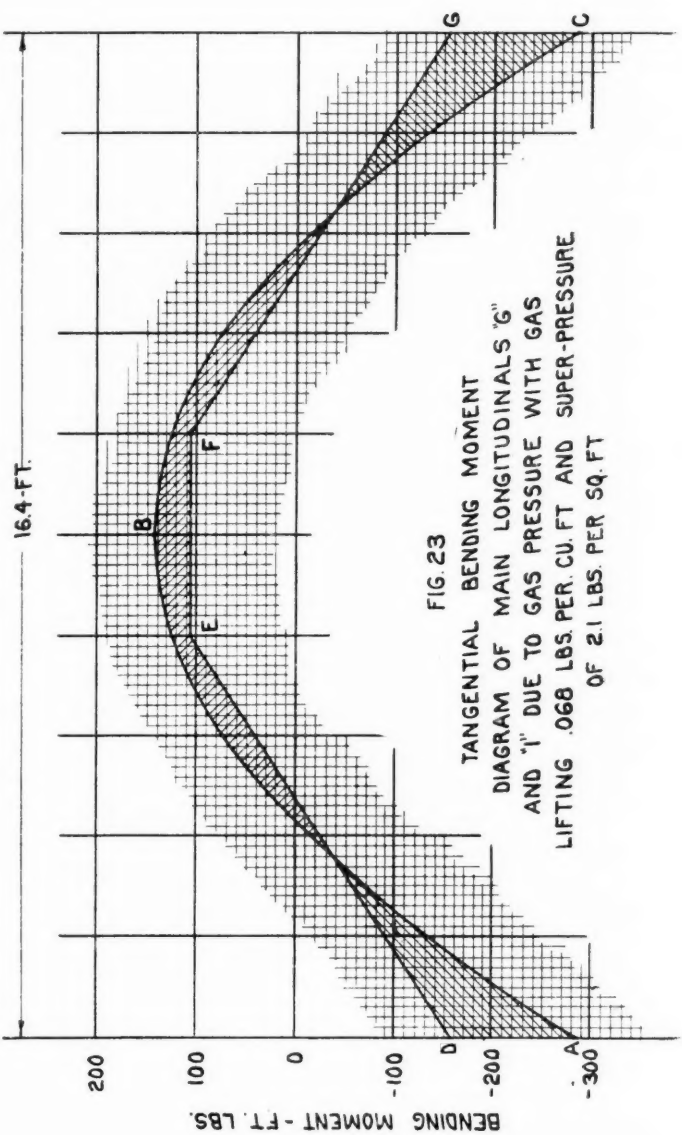


FIG. 23

TANGENTIAL BENDING MOMENT DIAGRAM OF MAIN LONGITUDINALS "G" AND "I" DUE TO GAS PRESSURE WITH GAS LIFTING .068 LBS. PER CU. FT AND SUPER-PRESSURE OF 2.1 LBS PER SQ. FT

TABLE 6.

TENSIONS IN SECONDARY SHEAR WIRES AND VERTICAL COMPONENTS OF FORCES IN FRAME SPACE 100-105.

Wire.	Tensions. Initial. lbs.	Final. lbs.	Change. lbs.	$\sin\phi$	$\cos\theta$	Vertical component of change of tension. lbs.
A-100 to C-105 ...	150	95	−55	.7638	.2462	10.4
C-100 to A-105 ...	95	80	−15	.7638	.2462	−2.8
C-100 to E-105 ...	185	45	−140	.7638	.6788	72.5
E-100 to C-105 ...	120	95	−25	.7638	.6788	−13.0
E-100 to G-105 ...	145	20	−125	.7638	.9469	90.4
G-100 to E-105 ...	100	120	20	.7638	.9469	14.5
G-100 to I-105 ...	140	45	−95	.7638	.9855	71.5
I-100 to G-105 ...	120	130	10	.7638	.9855	7.5
K-100 to I-105 ...	120	120	0	.7638	.7854	.0
					Sum.	251.0 lbs.
A-100 to C′-105	170	110	−60	.7638	.2462	11.3
C′-100 to A-105	100	50	−50	.7638	.2462	−9.4
C′-100 to E′-105	150	90	−60	.7638	.6788	31.1
E′-100 to C′-105	140	115	−25	.7638	.6788	−13.0
E′-100 to G′-105	145	50	−95	.7638	.9469	68.8
G′-100 to E′-105	135	130	−5	.7638	.9469	−3.6
G′-100 to I′-105	120	0	−120	.7638	.9855	90.2
I′-100 to G′-105	140	170	30	.7638	.9855	22.6
K′-100 to I′-105	140	125	−15	.7638	.7854	9.0
					Sum.	207.0 lbs.

SECTION VII.

Effect of Tension in the Outer Cover

The calculation of the radially inward loads on the longitudinals from the tension in the outer cover is a problem of a similar nature to the calculation of the outward loads from the gas pressure acting on the netting, and the same methods will be applied to it. The cover is assumed to have a constant tension all around the circumference of the cross-section so that there are no tangential loads to be considered.

Let β equal half the exterior angle between adjacent sides of the hull (Fig. 24). When there is no difference between the air pressure inside and outside of the hull, and the cover is drawn tight with a tension t per unit width of the cloth, it will lie in a straight line between successive longitudinals, and will put upon the girders a uniform running load equal to $2t \sin\beta$. In the ZR.1 $\beta = 14^\circ\ 15'$ and $\sin\beta = .2460$.

The tension produced in the outer cover by a given difference between the external and internal air pressures cannot be computed exactly because of the lack of definite stress/strain relations in doped fabric. Let it be assumed that a difference of pressure equal to .5 in. of water (2.6 lbs./ft.^{2}) produces a bulge of

3 in. in the outer cover in a panel 9.76 ft. wide. Then the angle $\gamma = 4e/c = (4 \times .25)/9.76 = .1024$ radians $= 5^\circ\ 52'$.

$$\psi = \pi/2 + \beta \pm \gamma$$
$$= 90^\circ + 14^\circ\ 15' \pm 5^\circ\ 52'.$$

The tension is given by $t = pc^2/8e = (2.6 \times 9.76^2)/(8 \times .25) = 124$ lbs./ft.

If the excess of pressure is external, γ is positive and $\psi = 110^\circ\ 7'$, and the radial load $= 2 \times 124 \times \cos 110^\circ\ 7' = 85.3$ lbs./ft. inward, and the maximum compressive stress produced in the main longitudinals is 9900 lbs./in.^{2} in the base channels at the joints with the transverses. If the excess pressure is internal, γ is negative, $\psi = 98^\circ\ 23'$, and the radial load $= 2 \times 124 \times \cos 98^\circ\ 23' = 36.2$ lbs./ft. inward.

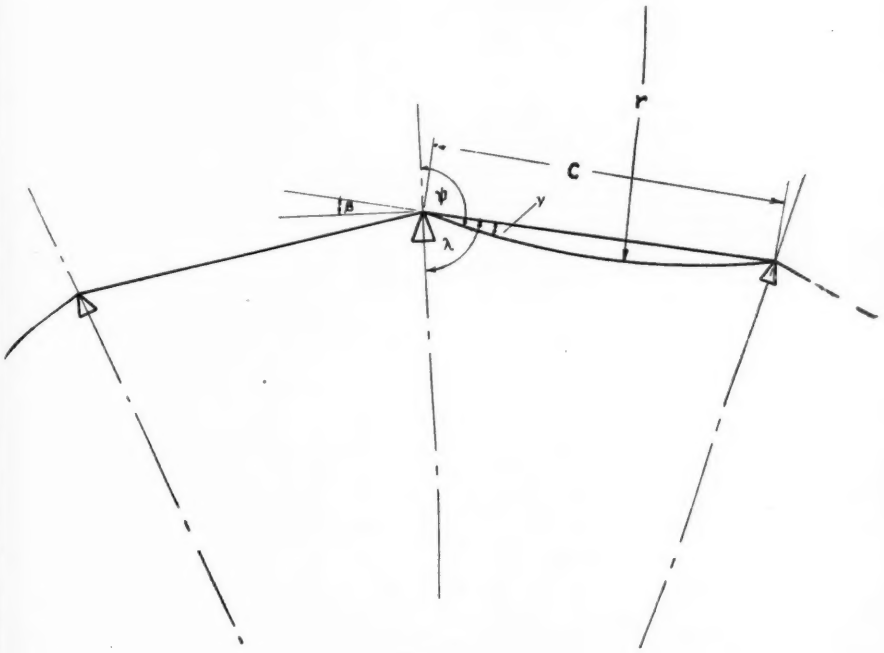


FIG. 24.
Illustrating load on longitudinals from outer cover.

From these examples may be seen the importance of preventing appreciable differences between the internal and external air pressure; and it is also evident that excess of external pressure is especially to be avoided. The load upon the longitudinals is always inward, unless the cover bulges out beyond the circumscribing circle through the apeces of the longitudinals, and this could never occur unless the cover were extraordinarily slack.

If the altitude of a rigid airship should change by even such a small amount as the ship's own diameter without passage of air into or out of the hull, the difference between the internal and external air pressures would exceed the difference between the gas pressures at the top and bottom of the hull. The

necessity of adequate openings for the flow of air is therefore apparent. The rate of flow through the openings is given by the expression :—

$$v = a\sqrt{2gp/\rho},$$

where v = the volume of air passing per second.
a = the total area of the openings.
g = the acceleration of gravity.
p = the difference between the external and internal air pressures.
ρ = the weight of the air per unit volume.

As an example, find the area of openings required to prevent p exceeding 0.5 lb./ft.^{2} in an airship of 2,000,000 cu. ft. volume, descending at 20 ft./sec., given $\rho = .072$ lbs./ft.^{3}, and the rate of change of density is 3.3 per cent. per 1,000 feet altitude.

$$v = 20/1000 \times .033 \times 2{,}000{,}000$$
$$= 1320 \text{ ft.}^3/\text{sec.}$$
$$a\sqrt{2 \times 32.2 \times .5/.072} = 1320$$
$$a = 62.5 \text{ ft.}^2$$

This is the area of openings required.

SECTION VIII.

Transverse Strength

The most severe condition for a main transverse frame is when a gas cell is deflated and the adjacent cells are full. The gas pressure on the ends of the inflated cells is taken by the transverse wiring, and the resulting tensions in these wires throw heavy compressive loads into the transverse girders. In normal conditions the gas pressures are approximately equal forward and aft of each frame, but there may be stresses of considerable magnitude from the concentrated loads applied to the bottom of the frames through the keel.

The stresses resulting from a deflated gas cell and from the keel loads will be considered separately; and finally, the effects of the combination of the two systems of loads will be investigated.

Owing to the extremely complex system of transverse wiring used in modern rigid airships (Fig. 25), it is impracticable to make an exact calculation of the manner in which the gas pressure loads will be apportioned among the wires. The authors have adopted the semi-empirical procedure of assuming that the entire system of transverse wiring may be replaced by a simple system of radial wiring, in which the cross-sectional areas of the wires are assumed to be twice as great as in the actual diametral wires of the airship. Calculations of the forces at the joints, resulting from this simplified system of wiring, have been found to agree very well with the recorded forces in gas cell tests of the British airships R.25, R.29 and R.38; and it will be shown that the results of the gas cell test of the ZR.1 also confirm the validity of the assumption.

Stresses Due to the Deflation of a Gas Cell

The equation of equilibrium for a wire carrying a transverse load is

$$-Td^2y/dx^2 = p,$$

where T = the tension in the wire.
y = the deflection of the wire from the straight line between the terminals at a distance x from a terminal.
p = the transverse load per unit length of the wire.

When the entire gas pressure on the end of a cell fully inflated without super-pressure is taken by radial wires secured to a centre ring which is permitted to move in the longitudinal direction only, the tension in the wires may be derived from the equation of equilibrium in the following manner:—

Let R = the length of the wire.
,, a = the cross-sectional area of the wire.
,, E = modulus of elasticity.
,, k = lift of the gas per unit volume.
,, δ = longitudinal movement of the centre ring.
,, d = half the angle between successive radial wires.

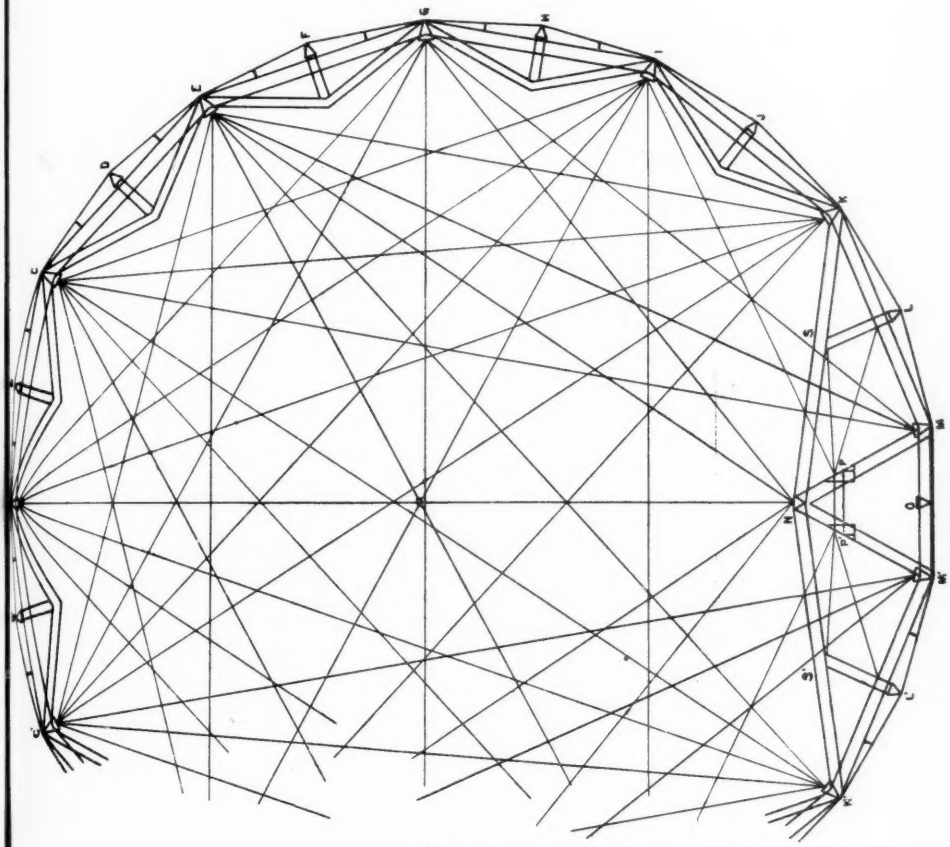


FIG. 25.
A main transverse frame of the ZR.1.

For the top vertical wire, the pressure on an element AB (Fig. 26) is $k(R+x)\,2x \tan \alpha dx$. From the differential equation of the equilibrium of the wire,

$$-Td^2y/dx^2 = p = 2k\,(Rx+x^2)\tan\alpha,$$
$$-Tdy/dx = 2k \tan\alpha\,(Rx^2/2 + x^3/3) + C,$$
$$-T\,(y-\delta) = 2k \tan\alpha\,(Rx^3/6 + x^4/12) + Cx.$$

When $x=R$, $y=0$, therefore:—

$$T\delta - 2k \tan \alpha \, (R^4/6 + R^4/12) = CR,$$

and

$$C = T\delta/R - \tfrac{1}{2}kR^3 \tan \alpha.$$

The extension of the wire is

$$\int_0^R \tfrac{1}{2} (dy/dx)^2 \, dx = \tfrac{1}{2} \int_0^R \{ (4k^2/T^2) \tan^2 \alpha \, (R^2x^4/4 + Rx^5/3 + x^6/9) + 4Ck \{ (\tan \alpha)/T^2 \} (Rx^2/2 + x^3/3) + C^2/T^2 \} \, dx,$$

$$= \tfrac{1}{2} [(4k^2/T^2) (\tan^2 \alpha) (153/1260) R^7 + k (\tan \alpha) (R^4/T^2) (T\delta/R - \tfrac{1}{2}k \tan \alpha \, R^3) + R/T^2 \, (T\delta^2/R^2 + \{ k^2R^6 \tan^2 \alpha \} /4 - T\delta kR^2 \tan^2 \alpha)].$$

Substituting for C above, this reduces to

$$(33/280) \, k^2R^7 (\tan^2 \alpha)/T^2 + (\delta^2/2R).$$

The extension of the wire also equals TR/Ea. Therefore

$$TR/Ea = (33/280) \, (k^2R^7 \tan^2 \alpha)/T^2 + \delta^2/2R.$$

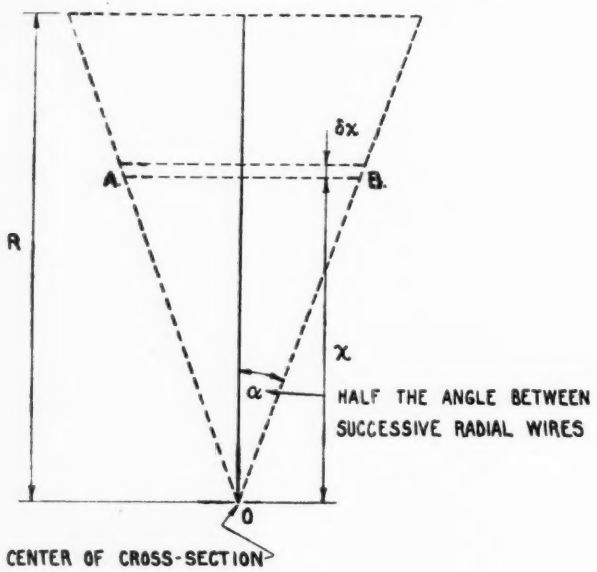


FIG. 26.

Gas pressure on upper vertical wire of a transverse frame.

For the horizontal wires, the pressure on an element is $2kRx \tan \alpha dx$, and proceeding in the same manner as before, the following equation is obtained:—

$$TR/Ea = (2/45) \, (k^2R^7 \tan^2 \alpha)/T^2 + \delta^2/2R.$$

For the bottom vertical wires, the pressure on an element is $2k \, (R-x) \, x \tan \alpha dx$, and proceeding as before:—

$$TR/Ea = (17/2520) \, (k^2R^7 \tan^2 \alpha)/T^2 + \delta^2/2R.$$

If a wire has an initial tension, T_0, before the transverse load is applied, the extension of the wire due to the transverse load when the terminals do not move is equal to $(T-T_0) \, R/Ea$, and the expressions derived for the tensions in the wires are modified by an added term T_0R/Ea on the right hand side.

If the initial tension is due to a change, ΔR, in the distance between the terminals of the wire, $T_o = Ea\Delta R/R$ and $T_o R/Ea = \Delta R$.

Equilibrium of forces at the centre ring requires that the ring move upward until the upward and downward forces at the ring are equal. Assuming that in the condition of equilibrium the tensions in the upper and lower vertical wires are equal, and designating the upward movement of the centre ring by ξ, the tension in these wires is given by :—

$$TR/Ea = (33/280)\,(k^2R^7 \tan^2 \alpha)/T^2 + \delta^2/2R - \xi,$$
$$= (17/2520)\,(k^2R^7 \tan^2 \alpha)/T^2 + \delta^2/2R - \xi.$$

Therefore

$$TR/Ea = (157/2520)\,(k^2 + R^7 \tan^2 \alpha)/T^2 + \delta^2/2R.$$

To find the maximum value of δ likely to occur in practice, it is assumed that the axial cable receives from the centre ring one-third of the total pressure of the end of the cell. In the ZR.1, with gas lifting .068 lbs./ft.^{3}, and no super-pressure, the mean pressure is 2.7 lbs./ft.^{2}, and the area of the end of the cell is 4510 ft.^{2}. Therefore the load on the axial cable is 4060 lbs. Its cross-sectional area is .1105 in.^{2}, and assuming the effective modulus of elasticity of steel cable is 17,000,000 lbs./in.^{2}, the stretch of the cable in 600 ft. is 1.30 ft. This may be taken as the maximum movement which could occur in the event of a gas cell becoming deflated. In the ZR.1 gas cell test, the cable was only 32.8 ft. long, or 16.4 ft. each way from the centre of the cell, and by calculation $\delta = .033$ ft. It will be shown that even when $\delta = 1.3$ ft., its effect upon T is small.

In the gas cell test of the ZR.1 the given quantities were :—

$R = 38.2$ ft.
$E = 30{,}000{,}000$ lbs./in.^{2}.
$k = .063$ lbs./ft.^{3}.
$\alpha = 14^\circ\ 15'$.
$\tan \alpha = .2540$.

The diametral wires consist of pairs of §9 gauge wires, each having a cross-sectional area of .00817 in.^{2}. The area of equivalent radial wires in the simplified system is therefore given by $a = 4 \times .00817 = .03268$ in.^{2}.

Let $\delta = 0$, then for the vertical wires $T = 3660$ lbs., and for the horizontal wires, $T = 3260$ lbs. When $\delta = 1.30$ ft., the magnitudes of T for the vertical and horizontal wires are, respectively, 3845 lbs. and 3460 lbs. From these figures it may be seen that the effect of the maximum longitudinal movement of the centre ring is quite small, and in a test of a single gas cell it is entirely negligible.

The radial and tangential forces at the joints of frame §100 are shown in Table 1. The radial forces may be compared with the calculated values of T in the equivalent radial wires when $\delta = 0$. The forces at the top and bottom joints agree well with the calculation, but at the mid-height the recorded radial forces are about 22 per cent. less than calculated.

No account has been taken of the decrease in the wire tension resulting from the contraction of the frame under the loads. Table 2 shows the observed movements of the joints. The effect of these movements is equivalent to giving the wires negative initial tensions corresponding to the movements.

Neglecting δ, the tension in the vertical wires is given by

$$TR/Ea = (157/2520)\,(k^2R^7 \tan^2 \alpha)/T^2 + \Delta R,$$

where ΔR is the mean radial movement of the top and bottom joints, considered as negative when the movement is inward. The tension in the horizontal wires is given by a similar expression in which the fractional multiplier is 2/45 and ΔR is the radial movement of the joints at midheight of the hull.

From Table 2 the mean radial contractions were .0083 ft. vertically and

.04 ft. horizontally. Using these values of ΔR, it is found that $T = 3580$ lbs. and 2950 lbs. in the vertical and horizontal directions respectively. The agreement with the observed forces is quite good.

The compressive forces in the truss elements of the main transverse frames are such as to give equilibrium with the forces applied to the joints by the wires. The truss AC is the most heavily stressed one, and the total compressive force in it is equal to $T/2 \sin \alpha$. When $T = 3785$ lbs. the end load on the truss is 7700 lbs.

Stresses Due to the Concentrated Loads at the Keel

The concentrated loads applied by the keel to the bottom of the main transverses are balanced by lift forces applied to the joints of the frame, partly as radial and partly as tangential forces. The lift which is received as shear from the next frames forward and aft is wholly tangential. The calculation of the loads in the frame can be much simplified by assuming that all the lift forces are tangential; and the resulting error is upon the safe side, because the radial forces, being outward, slightly offset the inward forces from the transverse wiring.

From the shear theory, tangential lift forces applied to the joints of the frame are proportional to $\sin u$ where u, as in previous sections of this paper, is the angle between the longitudinal plane through the joint and the longitudinal vertical plane, and the effect of variations in the sizes of the wires is neglected. The vertical component of the tangential force is proportional to $\sin^2 u$. It follows that if W is the concentrated load applied at the bottom of the frame, the tangential force at any joint equals $W \sin u / \Sigma \sin^2 u$, and the vertical force equals $W \sin^2 u / \Sigma \sin^2 u$, where the summation is taken all around the frame. The calculations of $\Sigma \sin^2 u$ and the tangential and vertical forces when $W = 6{,}500$ lbs. are shown in Table 3. This is the heaviest concentration of loads which occurs in the keel of the ZR.1.

Assuming that the actual wiring of the frame is replaced by the equivalent system of radial wiring previously considered, the forces at the joints and in the girders are as shown in Table 4.

When the condition of a concentrated load in the keel is combined with a deflated gasbag, the initial tension T_0 is taken as determined by the concentrated load. As an extreme case, assume $W = 6500$ lbs. and $k = .068$ lbs./ft.^{3}. In the radial wire AO, $T_0 = 1451$ lbs., $T = 4000$ lbs., and the compressive load in the truss AC = 8120 lbs.

The mean failing load of three transverse truss elements tested at the Bureau of Standards was 9,280 lbs., so that this extreme case is nearly approaching the limit of the strength of the frame. In practice no such great load would be carried in the keel at a frame adjacent to a deflated gas cell.

TABLE 1.

TENSIONS IN THE TRANSVERSE WIRES AT FRAME 100 IN THE GAS CELL TEST OF THE ZR.1.

Wire.	Tension, lbs. Initial.	Final.	Difference.	Radial Component.	Tangential Component.
AG′	65	430	365	247	269
AI′	55	470	415	348	226
AK′	45	555	510	481	170
AN	45	870	825	825	0
AN	—	850	850	850	0
AK	—	555	555	523	−185
AI	65	430	365	306	−199
AG	80	420	340	230	−251
				3810	30

TABLE 1.—*Continued.*

Wire.	Tension, lbs. Initial.	Final.	Difference.	Radial Component.	Tangential Component.
CE′	45	380	335	226	247
CG′	65	630	565	474	308
CI′	75	490	415	391	138
CK′	50	650	600	598	47
CK′	50	650	600	598	47
CM	—	585	585	552	−195
CK	—	490	490	411	−267
CI	50	280	230	156	−170
				3406	155
C′E	—	310	310	210	228
EE′	90	630	540	453	294
EI′	75	740	665	663	52
EI′	80	740	660	658	52
EK′	50	450	400	395	−66
EN	45	495	450	411	−183
EM	—	425	425	356	−232
EK	—	290	290	196	−214
				3342	−69
AG	80	420	340	230	251
C′G	98	650	552	463	300
GG′	95	790	695	692	55
GG′	85	790	705	702	55
GN	50	385	335	276	−190
GS	50	340	290	196	−214
				2559	257
CI	50	280	230	156	170
AI	65	430	365	306	199
C′I	70	590	520	490	174
E′I	75	730	655	652	51
E′I	75	730	655	652	51
II′	160	300	140	128	−57
IP	105	125	20	13	−15
				2397	573
EK	—	290	290	196	214
CK	—	490	490	410	267
AK	—	555	555	523	185
C′K	—	710	710	707	56
C′K	—	710	710	707	56
E′K	50	430	380	375	−63
				2918	715

TABLE 2.

OBSERVED RADIAL MOVEMENTS OF THE JOINTS OF MAIN FRAMES DUE TO COMPLETE INFLATION OF ONE CELL WITH GAS LIFTING .063 lbs./ft.^{3}.

Joint.	Frame 100. inches.	Frame 110. inches.	Joint.	Frame 100. inches.	Frame 110. inches.
A	−.200	−.180			
B	−.395	−.385	B′	−.350	−.342
C	−.204	−.210	C′	−.203	−.180
D	−.555	−.545	D′	−.495	−.493
E	−.512	−.510	E′	−.440	−.462
F	−.612	−.632	F′	−.550	−.488
G	−.343	−.330	G′	−.320	−.300
H	−.575	−.585	H′	−.588	−.532
I	−.510	−.530	I′	−.470	−.470
J	−.395	−.385	J′	−.450	−.438
K	−.105	−.110	K′	−.050	−.120
L	−.100	−.090	L′	−.100	−.125
M	−.015	−.040	M′	−.025	0
N	−.035	0			

TABLE 3.

CALCULATION OF LIFT FORCES AT JOINTS.

Joint.	u. Degrees.	sin u.	sin^2 u.	Tan. Force. lbs.	Vert. Force. lbs.
A	0	0	0	0	0
C	28.5	.4772	.228	492	235
E	57.0	.8387	.703	864	724
G	85.5	.9969	.994	1027	1024
I	66.0	.9135	.835	940	860
K	37.5	.6088	.371	627	382
M	9.0	.1564	.0245	161	25
			3.1555 × 2		3250 × 2
			Σ sin^2 u = 6.311		= 6500 lbs.

TABLE 4.

FORCES IN A MAIN TRANSVERSE FRAME OF THE ZR.1 DUE TO A CONCENTRATED LOAD OF 6,500 lbs. IN THE KEEL.

Equivalent radial wire.	Tension. lbs.	Truss.	Compressive load, lbs.
OA	1451	AC	2944
OC	1326	CE	2436
OE	980	EG	1545
OG	500	GI	485
OI	0	IK (tension) ...	485

NOTE.—The centre ring is designated by 0.

SECTION IX.

Factors of Safety and Comparative Strength

Up to this point the various loads upon the structure have been considered separately. The choice of loading conditions to be combined in estimating the factors of safety is a matter of judgment, and certain comparisons are to be ruled out as representing extreme conditions in which no commander should ever involve his ship. For example, analyses of the bending moments in the deflated gasbag condition and when the airship runs at full speed into a vertical gust show that these conditions may produce at frame 90 sagging bending moments of 277,000 m. lbs. and 318,000 m. lbs., respectively, and the total bending moment is 595,000 m. lbs., with a compressive load of 13,400 lbs./in.^{2} in the top longitudinal. In addition there may be a compressive stress of 11,000 lbs./in.^{2} in the apex channel from gas pressure and secondary bending of the longitudinals, making the total compressive stress 24,400 lbs./in.^{2} in the apex channel at the joint. This portion of the girder is double latticed, and the failing stress is shown by the Bureau of Standards' test to be about 27,000 lbs./in.^{2}, so that in this extreme condition the factor of safety is only 1.11.

With a deflated gas cell an airship should never be driven at full speed even in still air, so that this condition may be considered as too severe for a fair estimate of the real factor of safety. In the design of the ZR.1, theoretical factors of safety have been based upon two conditions which may be compared with the standard hogging and sagging conditions employed by naval architects in calculating the strength of sea-going ships. These conditions are defined as follows:—

(*a*) *Sagging.*

The airship is assumed to be carrying the normal full load, giving the bending moment represented by curve A of Fig. 15, but with an excess of weight which is compensated by a positive angle of pitch at 6° at a speed of 68 ft./sec., giving the sagging bending moment shown by curve C of Fig. 18. The total sagging bending moment at frame 90 is 187,000 m. lbs., and the change of shear at this frame is 6,558 lbs. The cells are assumed to be full of gas lifting .068 lbs./ft.^{3} with a super-pressure of 2.1 ft.^{2}. Since the upper longitudinals are the critically loaded ones in this condition, the outer cover is assumed to be slack, so as not to count upon a probable but uncertain reinforcement from the cover against the gas pressure. The resultant stresses in the upper longitudinals at frame 90 are summarised in Table 1.

(*b*) *Hogging.*

The airship is assumed to be in the modified full load condition represented by curve B of Fig. 15, but with an excess of buoyancy compensated by a negative angle of pitch at 6° at a speed of 68 ft./sec., giving hogging bending moments shown by curve C of Fig. 18. At frame 90 the total hogging bending moment due to static and dynamic forces is 336,680 m. lbs. and the change of shear is 5,588 lbs. The gas pressure is assumed to be the same as in the sagging condition; but in this case the lower longitudinals are the critically loaded ones, and as the gas pressure throws very little load upon them, it is quite possible for the inward load from the outer cover to be much more severe than the gas pressure load. For this reason it is assumed that the cover puts a radially inward load of 20 lbs./ft. on the longitudinals. The stresses in the lower longitudinals in this condition are also summarised in Table 1.

Assuming that the crippling stresses in the longitudinals are 27,000 lbs./in.^{2} the theoretical factors of safety in these sagging and hogging conditions are 1.68 and 1.82 respectively.

As was shown in Section VIII., the factor of safety in the transverse frame in the deflated gasbag condition, without any concentration of loads in the keel near the deflated bag, is 1.21. In practice, if it is necessary to carry concentrated loads at a frame sustaining the end pressure resulting from the complete deflation of a gas cell, the neighbouring cell would also be partially deflated, so that this may be taken as the minimum factor of safety transversely.

These factors of safety may appear low in comparison with accepted practice in other branches of engineering; but it will be shown that the ZR.1 compares favourably in strength with successful German and British airships, and it is always by the criterion of successful practice that the adequacy of the factors of safety in any branch of engineering must be judged.

Comparative Longitudinal Strength of the ZR.1 and other Rigid Airships

The design of the airship ZR.1 has been derived in the main from the German airship L.49. The design, arrangement and sizes of structural members have followed the L.49 with few exceptions; and the important modifications have been in accordance with the German airship L.72, and have been towards an increase of strength.

The longitudinal strength of the ZR.1 may be compared with other modern rigid airships under six headings as follows:—

(*a*) Moment of inertia of the cross-sections of the hull.
(*b*) Sizes of the diagonal shear wiring.
(*c*) Static bending moments.
(*d*) A coefficient of relative aerodynamic stress.
(*e*) Power of control surfaces.
(*f*) Areas of fins.

These six bases of comparison will be considered in order.

(*a*) *Moment of inertia.*

Table 2 shows the comparative moments of inertia of the ZR.1 and other rigid airships. The calculations are based upon the areas of the longitudinals without regard to any of the refinements of the bending theory discussed in Section IV. For comparison purposes this seems a logical method of procedure. The airships compared are all of similar type, and the modifications introduced in the longitudinal strength by correction factors, or Prof. Hovgaard's fictitious bars, could scarcely make more than 1 per cent. difference in their comparative strengths.

It may be noted that in the ZR.1 special care has been taken to maintain the strength towards the bow and stern; and in many cases the full sectional area of the longitudinals have been maintained to nearly the extreme bow and stern, although the corresponding members of the German airships have been slightly reduced in area.

The main transverses of the ZR.1 are 10 metres apart with one intermediate transverse, as in the L.49 and R.33, instead of at 15 metres apart with two intermediates, as in the L.72 and R.38. The amount of support which an intermediate transverse gives to the longitudinals is not entirely definite and beyond dispute. Undoubtedly the arrangement of the main and intermediate transverses in the ZR.1 and L.49 lends greater strength to the longitudinals than the arrangement of the L.72 and R.38, against both end loads and gas pressure, although the difference may be small.

A strength feature of the ZR.1, borrowed from the L.72, is that shear wires are secured to the joints between the main transverses and the secondary longitudinals, making the latter members more effective in the primary strength of the

hull than in the L.49, where no shear wires are attached to them. Another feature in the ZR.1, also borrowed in part from the L.72, is the use of larger intermediate longitudinals in the lower than in the upper part of the hull.

(*b*) *Strength of shear wires.*

From the shear theory of longitudinal strength, the tension produced in a shear wire by parallel movement between the transverse frames to which it is attached is proportional to $a \sin\phi \cos\theta/b$. The component of the tension in the direction of the shearing force is equal to the tension multiplied by $\sin\phi \cos\theta$; and when b is constant, the length of the wire is inversely proportional to $\cos\phi$. It follows that a measure of the effectiveness of a shear wire in resisting parallel movement between the transverses due to the shearing forces in the hull is given by $a \sin^2\phi \cos^2\phi \cos^2\theta$. Table 3 shows the comparative magnitudes of $\Sigma a \sin^2\phi \cos^2\phi \cos^3\theta$ for the ZR.1 and other airships.

(*c*) *Static bending moments.*

The use of six power cars instead of four gives better distribution of the fixed weights than in the L.49 and R.33, as may be seen by comparison of the static bending moments in the light condition, shown in Table 4. The static bending moment in the normal full load condition is small in all the airships compared.

(*d*) *Coefficient of aerodynamic stress.*

In airships in which the shape of the hull and the relative effectiveness of the fins and control surfaces are approximately the same, the transverse aerodynamic forces due to manœuvring or flying in gusty weather are shown by Dr. Munk's theory to be proportional to the square of the diameter of the hull, the density of air, and the square of the speed. The resulting bending moments are proportional to the magnitude of the forces multiplied by the length of the ship, or in mathematical symbols:—

$$M \sim LD^2\rho V^2$$

$$\text{Also } f \sim M/AD$$

Where M = bending moment due to aerodynamic forces.
L = length of hull.
D = diameter of hull.
ρ = density of the air.
V = speed of the airship.
f = maximum stress in the longitudinals due to M.
A = total sectional area of the longitudinals in the cross-section of the hull.

Combining these two expressions,

$$f \sim LdV^2\rho/A.$$

Let Q_g = a coefficient of stress due to aerodynamic forces, proportional to f, then

$$Q_g = LDV^2\rho/1000\,A$$

Or if regard is given to Prof. Hovgaard's fictitious bars, but without correction factors, the divisor in this expression becomes $A + A_f$.

Since the magnitude of the coefficient depends so largely upon the speed, the validity of conclusions drawn from application of the coefficient to various airships depends upon the reliability of the speed data. In Table 5 the recorded speeds of a number of German and British airships of modern form are compared, and the probabilities of the accuracy of these speeds are checked by the Admiralty constants.

The values of Q_g for the ZR.1, based on Admiralty constants of 3,000 at a height of 6,000 ft. altitude and 2,825 at sea-level, are shown in Table 6. It may be noted that theoretically the Admiralty constants vary inversely as the cube root of the density of the air. Comparison of the values of Q_g, having a special regard to the L.49, the airship on which the design of the ZR.1 is mainly based, is believed to show that with a horse-power of 750, corresponding to a speed of 51.75 knots, and $Q_g = 1,115$ at 6,000 ft. altitude, there will be an ample margin of safety against excessive dynamic stresses in the ZR.1. Experimental data from the trials may show that higher speeds can be safely attained. This speed of 51.75 knots has been used for the calculations of the aerodynamic forces in the ZR.1, with appropriately reduced speeds at fixed angles of pitch, as described in Section III.

(e) Power of control surfaces.

One of the aerodynamic conditions to be considered is sudden application of the rudders or elevators when the airship is proceeding along a path parallel to its longitudinal axis. The stresses in the longitudinals in this condition are proportional to the moment produced by the control surface about the centre of buoyancy, and inversely proportional to the section modulus of the hull. A coefficient of stress, Q_e, based on these proportionalities, is shown in Table 7. The forces applied to the stern by the application of elevators are calculated from wind tunnel data.

The high value of Q_e for the R.38 is noteworthy. At 54 knots and 2,500 ft. altitude, the conditions under which she broke, the value would still be 1,520, or considerably higher than for any other airship.

(f) Fin areas.

It may be expected that when flying in gusty winds the aerodynamic forces on an airship will be to some extent a function of the area of the fins, the larger the fins the greater the stresses, although probably by no means in direct proportion. A comparative coefficient, Q_f, of stress due to the fins in rough weather may be expressed in the form

$$Q_f = LV^2F/Z$$

Where F is the area of the fins and Z the section modulus of midship section of the airship. Values of this coefficient are shown in Table 8, based on the areas of the horizontal fins.

TABLE 1.

SUMMARY OF STRESSES, lbs./in.², IN UPPER AND LOWER LONGITUDINALS OF THE ZR.1 AT FRAME 90 IN THE EXTREME SAGGING AND HOGGING CONDITIONS.

Longitudinal.		Sagging. A	Sagging. B	Sagging. C	Hogging. K	Hogging. L	Hogging. M
Initial stress ...		−1,175	−1,175	−1,280	−885	−885	−1,245
Primary bending...		−3,620	−3,540	−3,500	−5,250	−6,120	−8,250
* Gas pressure	Apex	−8,455	−2,780	−9,090	−155	1,975	—
	Base	5,225	1,498	4,557	−1,851	−2,782	−2,695
Outer cover	Apex	—	—	—	3,690	5,600	−2,695
	Base	—	—	—	−2,320	−4,120	—
Secondary stress	Apex	−2,485	−1,230	−2,225	1,715	1,292	1,960
	Base	1,540	965	1,400	−1,075	−953	−1,432
Total stress	Apex	−15,735	−8,725	−16,095	−885	1,862	−10,230
	Base	1,970	−2,252	1,177	−11,381	−14,860	−13,622

NOTE.—* Gas pressure effect includes rise of intermediate transverse.

TABLE 2.

TOTAL AREAS OF LONGITUDINAL GIRDERS, MOMENTS OF INERTIA AND SECTION MODULI OF MIDSHIP SECTIONS OF RIGID AIRSHIPS.

Airship	ZR.1	L.49	L.72	R.33	R.38
Total areas of longitudinal girders, mm.^{2}	4,960	4,830	4,872	5,216	4,803
Diameter of midship section, m.	24.0	24.0	24.0	24.0	26.0
I of midship section about horizontal axis, m.^{4}	0.3679	0.3543	0.3625	0.3869	0.4332
Section modulus about horizontal axis, m.^{3} ...	0.02733	0.0265	0.02721	0.02933	0.0302
Do. ft./in.^{2}	139.0	135.0	138.6	144.0	154.0

TABLE 3.

EFFECTIVENESS OF SHEAR WIRES AT MIDSHIP SECTION AGAINST SHEAR IN THE LONGITUDINAL VERTICAL AND HORIZONTAL PLANES.

Airship.	ZR.1	L.49	L.72	R.33	R.38
Total areas of shear wires, Σa, in.^{2} ...	0.07574	0.07516	0.06903	0.08490	0.12038
$\Sigma a \sin^2 \phi \cos \phi \cos^2 \theta$ in vertical bending ...	0.011508	0.011580	0.011425	0.012985	0.026268
Do. in horizontal bending	0.008507	0.00814	0.009710	0.011018	0.015610

TABLE 4.

COMPARISON OF MAXIMUM STATIC BENDING MOMENT OF RIGID AIRSHIPS IN THE LIGHT AND HEAVY CONDITIONS.

Airship.	ZR.1	L.49	R.33	R.38
Maximum B.M. in light condition, m. lbs.	188,500	211,000	236,000	166,000
Do. in heavy condition, m. lbs. ...	96,750	48,400	62,500	−68,500
f max., light, lbs./in.^{2}	4,450	5,140	5,190	3,540
f max., heavy, lbs./in.^{2}	2,195	1,180	1,370	−1,460

NOTE.—Positive bending moment denotes hogging condition. Values of the fibre stress, f max., are based on the section moduli given in Table 2.

TABLE 5.

REPORTED SPEEDS AND ADMIRALTY CONSTANTS OF RIGID AIRSHIPS AT 6,000 ft. ALTITUDE.

Airship.	Displacement. Cu. ft.	Tons.	$Tons^{2/3}$.	H.P.	Authority for speed.	Speed. Knots.	Admiralty constant.
L.33	2,140,000	61.0	15.5	1200	Munk ...	54.7	2120
L.36	,,	,,	,,	,,	Jaray ...	55.7	2240
L.43	,,	,,	,,	1000	Munk ...	52.6	2260
,,	,,	,,	,,	,,	Jaray ...	53.9	2425
L.44	,,	,,	,,	,,	Munk ...	55.6	2665
,,	,,	,,	,,	,,	Jaray ...	56.0	2730
L.45	,,	,,	,,	,,	N. B. ...	56.9	2860
L.46	,,	,,	,,	,,	Munk ...	56.5	2800
L.49	,,	,,	,,	,,	N. B. ...	54.4	2500
,,	,,	,,	,,	,,	Jaray ...	58.1	3040
L.57	2,640,000	75.3	17.8	,,	Munk ...	56.0	3140
,,	,,	,,	,,	,,	Jaray ...	55.6	3060
L.70	2,400,000	68.5	16.7	1680	Munk ...	67.1	3000
,,	,,	,,	,,	,,	Jaray ...	70.6	3500
R.33	2,185,000	62.4	15.7	1040	R. & M. 668	52.6	2200
R.38	2,960,000	84.5	19.2	1760	Trials ...	61.4	2525

NOTE.—Munk: "The Drag of Zeppelin Airships," N.A.C.A. Report, No. 117.
Jaray: "Studien zu Entwicklung der Luftfahrzeuge," Zeitschrifft für Flugtechnick, 15 June, 1920.
N.B.: Notes found on German airships.

TABLE 6.

VALUES OF THE STRESS COEFFICIENT, Q_g.

Air ship.	Length L ft.	Diam. D ft.	H.P.	Speed V knots.	Altitude ft.	Density of air ρ lbs./ft.^{3}	Area of longitudinals Ain.^{2}	Area of Fictitious Bars $A_f$in.^{2}	$Q_g = \frac{LDV^2\rho}{1000\,(A+A_f)}$
L.49	643	78.7	1000	56.5	6000	.0639	7.50	.485	1232
L.49	643	78.7	1200	56.5	0	.07633	7.50	.485	1473
L.70	692	78.7	1680	67.1	6000	.0639	7.55	.387	1978
R.33	643	78.7	1040	52.6	6000	.0639	8.10	.515	1040
R.33	643	78.7	1250	52.6	0	.07633	8.10	.515	1243
R.38	695	85.4	1760	61.4	6000	.0639	7.45	.642	1770
R.38	695	85.4	2100	61.4	0	.07633	7.45	.642	2105
R.38	695	85.4	—	54.0	2500	.0709	7.45	.642	1520
ZR.1	676	78.7	750	51.75	6000	.0639	7.70	.482	1115
ZR.1	676	78.7	900	55.0	6000	.0639	7.70	.482	1260
ZR.1	676	78.7	688	47.4	0	.07633	7.70	.482	1115
ZR.1	676	78.7	823	50.3	0	.07633	7.70	.482	1260

TABLE 7.

VALUES OF COEFFICIENT, Q_e, OF STRESS DUE TO SUDDEN TURNING OF THE ELEVATORS TO 15° ANGLE AT 6,000 ft. ALTITUDE.

Airship.	ZR.1	L.49	L.70	R.33	R.38
Length, ft.	676	643	692	643	695
Section modulus, Z, ft./in.^{2} ...	139.0	135.0	138.6	144.0	154.0
Speed, knots	51.75	56.5	67.1	52.6	61.4
Moment of force on elevators about C.B. ft./lbs. × 1,000...	1,075	1,055	1,480	1,052	2,000
Source of wind tunnel data ...	Bu. Aero. SA.120	Bu. Aero. SA.82a	Bu. Aero. SA.82a	R. & M. 361	T. 1,470
Q_e = moment of force/section modulus	7,740	7,820	10,700	7,300	13,000

TABLE 8.

VALUES OF COEFFICIENTS, Q_f, OF STRESS DUE TO FINS.

Airship.	ZR.1	L.49	L.70	R.33	R.38
Length, ft.	676	643	692	643	695
Section modulus, Z, ft./in.^{2} ...	139.0	135.0	138.6	144.0	154.0
Speed, knots	51.75	56.5	67.1	52.6	61.4
Area of horizontal fins, F, ft.^{2}...	2,510	1,890	2,160	1,850	2,310
$Q_f = LV^2F/1000\,Z$	32,800	28,750	48,600	22,850	39,400

NOTE.—Stresses due to fins probably vary less rapidly than as Q_f.

SECTION X.

Summary

In Conclusion

The final judgment of the structural strength of a rigid airship rests ultimately upon experience which we cannot yet pretend to have acquired. However, in order to apply experience usefully there must be a theory and a method in order that the isolated facts of experience may arrange themselves to reveal a doctrine for the designer. Academically, if we know, first, the external forces to which the ship in service is subjected, and secondly, the stresses induced in the structural members of the ship by those external forces, we might assign with confidence the factors of safety which are usual in engineering structures. However, the matter will never be so straightforward as this.

Since the naval architect with his centuries of accumulated experience is still unable to calculate the precise effect on a vessel of a gale at sea, it seems certain that the force of wind and weather on an airship will remain equally unknowable. Consequently, we must follow the naval architect's method, which is substantially as follows:—

(*a*) Assume an arbitrary method for the estimation of external forces which will be applied to all ships, making allowance for speed, size, shape, etc.

(*b*) Employ standardised calculations, as precise as practicable, to obtain the stresses in the structure due to the external forces.

(*c*) Compare the results of standardised calculations to successful and unsuccessful experience to obtain factors of experience, or factors of safety. Such a factor of safety is in no sense a true margin of security, but does furnish a powerful method of comparison between some actual airship and a proposed design. It permits experience to be accumulated in useful form.

In this paper the authors have developed methods for estimating both the external forces and the stresses which are induced in the ship. The external forces are arbitrary to a great extent and the calculations of the stresses are approximate. However, it is believed that the external forces here estimated are proportional to what must actually occur, and include all the necessary variables to permit application of the methods of this paper to airships of different designs. Similarly, the calculations for the stresses in the structure of the airship, while approximate, are much more precise than the estimation of external loads, and no less so than is tolerated in the strength calculations for vessels. These calculations are also believed to include all the necessary variables to permit application to different designs.

The authors cannot too strongly urge the advantages to the art of the free dissemination of scientific and technical information in order that knowledge acquired item by item may serve to advance civilisation, as such knowledge becomes organised. The development of airships has been attended by the disasters inevitable in pioneer work. The pioneers who lost their lives in R.38 made the crucial test of the state of our knowledge at that time. We, who follow them, must proceed in all humility and with full appreciation of the lessons which their splendid sacrifice has made available.

In closing, it seems appropriate that the authors, all three of whom were associated with the late C. I. R. Campbell, R.C.N.C., in the early development of a theory of strength, should acknowledge their debt to him. Mr. Campbell visited the United States to confer upon the design of ZR.1, and the authors visited England to confer upon the design of R.38. The design of a rigid airship is the boldest and most exacting engineering adventure man has ever attempted. Mr. Campbell realised the known and the unknown elements of the problem, and dimly glimpsed the unknowable. From his painstaking examination of the meagre facts of experience available at the time, the authors were impressed by the necessity for continuous effort before the design of rigid airships could be placed on a sound basis, and his example and his advice guided them into the investigation which has led to the preparation of this paper. Consequently, the paper is dedicated to Mr. Campbell in the hope that it will be recognised as an extension of the work in which he was vitally interested and an effort to advance the art for which, with considerable knowledge of the uncertainties, he deliberately risked and gave his life.

APPENDIX I.

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Appendix II.

Sudden Application of the Rudders or Elevators

The view has been taken that the rudders could be put over so suddenly while proceeding on a straight course that the ship is subjected to a transverse force applied to the stern opposed only by the inertia of the ship against rotation. Naturally, as the rudders start to move, the ship starts to turn; but with a powerful steering gear the rudders could reach full throw before the ship had swung far from her course. The assumption that the rudder force is suddenly applied is then a limiting condition which must always be more severe than can be met in practice and, hence, is on the safe side.

The rotation of the ship due to the rudder force F must depend not only on the mass of the ship, but also on the mass of surrounding air disturbed by rotation, or on the so-called virtual mass. If the mass of the ship, as well as the additional mass of air, be assumed concentrated at the main frames, the shearing force, S, and bending moment, M, resulting from a transverse force, F, applied at the stern post may be calculated readily.

Let f = transverse inertia force at a main frame.
w = weight at a main frame.
a = transverse acceleration at a main frame.
l = distance of a main frame from centre of percussion.
b = distance between main frames.
g = acceleration of gravity.

Then $F = \Sigma f$, the summation being taken over the whole length and regard given to change of sign at centre of percussion, and

$$f = wa/g$$

where $a = Cgl$, and C is a constant determined by

$$F = C\Sigma wl.$$

The centre of percussion distance, say from the rudder post, is given by

$$R = \Sigma wl^2 / \Sigma wl.$$

$S = F - \Sigma f$, where the summation is taken from the bow or stern to the point where S is desired.

Finally $M = \Sigma Sb$, the summation being taken as for S.

From an examination of the above expressions, it will be seen that it makes no difference in S or M whether we use actual mass or virtual mass in computing w, provided we agree to distribute the additional mass due to the surrounding air along the ship's length in accordance with the known distribution of the actual weights. The virtual mass may be computed by the aid of Prof. H. Lamb's

formulæ, but the assumption of its distribution as w is distributed appears to be both reasonable and convenient. The result is a considerable simplification of the calculations, and gives us a straightforward, if somewhat arbitrary, calculation to apply to different ships to compare their relative strengths to withstand severe rudder (or elevator) action.

The magnitude of F and the location of the centre of pressure of aerodynamic forces on the tail surfaces might be obtained from wind tunnel tests on a model of the airship held with axis parallel to the wind stream, and with rudders set at some prescribed angle. In view of the rather arbitrary nature of the rudder action assumed and of the obvious advantage of having a simple calculation for purposes of comparison of different airships, it is recommended that F be applied at the rudder post or cruciform girder.

Wind tunnel experiments on a model of the ZR.1 showed that F would equal 3,060 lbs. when the elevators are turned 20°, and the speed is 88 ft./sec. The maximum bending moment in this condition is calculated to be 130,000 metre-pounds. It is instructive to investigate the angular acceleration corresponding to this value of F.

Let α = angular acceleration.
θ = angle of yaw in radians at time t after application of F.
ω = angular velocity at time t.

Then the work done by F is given by

$$FR\theta = \Sigma w\,(\omega l)^2 / 2g.$$

Since $\theta = 1/2\alpha t^2$ and $\omega = \alpha t$, therefore

$$F = \alpha \Sigma w l^2 / gR.$$

The value of $\Sigma w l^2$ for the ZR.1 in the full load condition, and omitting the additional apparent mass, is 420,900,000 m.²/lbs., and $R = 127.16$ m., with $F = 3{,}060$ lbs., $\alpha = .00905$ radians/sec.², or 0°.518/sec.².

It was stated that disregard of virtual mass is without importance in calculating bending and shear, and this must be so if the rudders are applied so suddenly that the ship cannot swing. However, in examining the possible rotation of the ship under the assumed rudder force the virtual mass enters directly. It is estimated that the virtual mass must be something less than twice the actual mass. If it be assumed that the use of virtual mass doubles the magnitude of $\Sigma w l^2$, the angular acceleration is reduced to 0°.26/sec.².

It is unlikely that such a large angular acceleration can occur in practice. If maintained for 10 seconds, the angular velocity at the end of that time would be 2.6 degrees/second, and the airship would have turned through 13°. The airship must attain a considerable angular motion before the rudders or elevators reach full throw, and the accelerating force is thereby reduced, with corresponding diminution of the shearing forces and bending moments.

Appendix III.

Aerodynamic Forces at a Fixed Angle of Pitch

Dr. Munk's theory deals with a "perfect" fluid, incompressible and without viscosity; and its flow is assumed to be irrotational and continuous. On these assumptions the following equation is derived for the distribution of the transverse force along the hull of an airship with circular cross-sections, and inclined at a fixed angle of yaw or pitch:—

$$dF/dx = f = (dS/dx)\,V^2 \tfrac{1}{2}\rho \sin^2 \alpha\,(k_2 - k_1).$$

Where F = the transverse force.
x = distance along the longitudinal axis of the airship.
S = cross-sectional area of the airship.
V = air speed.
ρ = density of the air in absolute units.
α = angle of yaw or pitch.

k_1 and k_2 = coefficients of the "additional" masses of air carried by the motion of the airship in the longitudinal and transverse directions, respectively. It is assumed that these coefficients have the same values as for an equivalent ellipsoid having the same ratio of length to the cube root of the volume as in the actual airship, and moving in a perfect fluid. The ratio length/diameter (L/D) for the equivalent ellipsoid is given by the equation :—

$$L/D \text{ (ellipsoid)} = \sqrt{(\pi/6)}\, L^3/\text{volume (airship)}.$$

Values of k_1 and k_2 for ellipsoids of various length/diameter ratios, as calculated by Prof. H. Lamb (N.P.L. Report R. and M. No. 623) are given in Table 1.

The basis of Dr. Munk's theory is that the additional mass of air carried along by the motion of the airship is the same around any short length of the hull as around an equal length of an infinite cylinder of the same cross-sectional area, with a correction factor equal to $k_2 - k_1$ to allow for the difference between an infinite cylinder and an ellipsoid of finite L/D. Where the cross-sectional area of the hull is increasing in the direction of the air flow, the additional mass of air is also increasing, and the resultant transverse air force on the hull from the windward towards the leeward side is such as to impart the momentum to the increasing mass of air. Conversely, where the cross-sectional area is decreasing, the diminishing momentum of the surrounding mass of air imposes a transverse force on the hull from the leeward towards the windward side.

The assumption of a perfect fluid with irrotational continuous flow involves a very considerable departure from reality; but the comparison (Fig. 17) with the N.P.L. experimental data shows that except on the after-body, or at large angles of pitch or yaw, the theory gives results in surprisingly good accordance with experiment. It is inferred, therefore, that over the greater part of the actual model, or airship, the departure from the streamline flow and the vorticity which occur in the real fluid have a relatively small effect on the distribution of the transverse forces on the hull. It is shown by the comparative curves (Fig. 18) that the discrepancy between the theoretical and observed distribution of transverse force along the after-body makes very little difference in the bending moments.

A curious feature of Dr. Munk's treatment of the problem is that the assumption of an ideal fluid is applied to the hull, but not to the tail surfaces. It is assumed that by setting the control surfaces to the appropriate angle, there is created near the stern an aerodynamic force giving equilibrium of moments with the aerodynamic forces on the bare hull. In modern rigid airships it is necessary to move the elevators downward if the angle of pitch is positive and upward if the angle is negative, in order to obtain the necessary force to give equilibrium. An airship appears, therefore, to be tail-heavy when actually heavy, and bow-heavy when light.

The exact position at which to assume the resultant of the forces created by the tail surfaces is a matter of judgment. In the ZR.1 it was taken to be 10 metres forward of the cruciform frame; and moving it 5 metres, with a corresponding change in its magnitude to maintain equilibrium of moments, was found to produce changes of only 5 per cent. in the maximum bending moment and the dynamic lift. It must not be supposed that the whole of the force assumed to be due to the tail surfaces is actually sustained by these surfaces. In reality, this

force represents the actual resultant force on the surfaces plus the difference between the actual and theoretical forces on the after-body of the bare hull. It is because the total forces on the after-body must give equilibrium with the forces on the fore-body, that the discrepancy between the actual and theoretical forces on the after-body of the bare hull is of small importance in calculating the bending moment. The discrepancy is largely absorbed in the assumed force near the stern, as may be seen by comparing the bending moment calculations based on Munk's formula and on the N.P.L. pressure distribution experiments.

Comparison of the curves (Fig. 17), giving the distributions of transverse force according to Munk's formula and the N.P.L. investigations, shows that the experimental results may be represented by transverse forces acting from the windward towards the leeward side of the model, superimposed upon the theoretical forces, and increasing from the bow towards the stern. Probably these additional forces are due chiefly to the turbulence (vorticity) of the air along the leeward side of the model, beginning near the bow, and increasing as the flow moves aft. This is just what would be expected in a real fluid compared with the ideal one. According to Dr. Munk's theory, there is no resultant force on the bare model at any angle of inclination, and not even a resultant moment when inclined at 90° to the direction of flow; whereas the actual bare model experiences a resultant transverse force, increasing with the angle of inclination. At small angles the resultant transverse force on the bare model is mainly due to the discrepancy between the actual and theoretical forces on the after-body, and this is shown to have little influence on the bending moments; but as the angle of pitch or yaw increases the discrepancy becomes greater, and extends over a larger portion of the model. The bending moments computed by the Munk formula and the N.P.L. curves of transverse force agree much more closely at 6° than at 12° pitch.

The transverse force due to the friction of the air upon the model is believed to be small at all angles of inclination, because the resultant forces and moments calculated by the N.P.L. from the transverse components of the normal air pressure on the model were found to be nearly identical with the forces and moments on the model as determined by the wind tunnel balance.

In calculating the forces upon the airship at a fixed angle of pitch, using either Munk's formula or the N.P.L. curve of transverse force, it is assumed that the resultant static force due to the excess weight or buoyancy (depending upon whether the angle of pitch is positive or negative) acts through the centre of buoyancy and gives equilibrium with the resultant of the aerodynamic forces upon the hull and tail surfaces. In practice it is not usually necessary to attain equilibrium of moments solely by setting the elevators. There is always a static righting moment partially offsetting the aerodynamic moment on the bare hull; and this static moment may be increased by moving the disposable loads forward. The static stability reduces the force to be applied by the elevators to give equilibrium of moments; and consequently reduces the dynamic lift. For ZR.1 at full load, the static righting moment for 6° pitch is about 90,000 metre-pounds, which amounts to 13 per cent. of the aerodynamic moment on the hull at this pitch and 68 ft./sec. velocity.

A very important point brought out by Munk's theory is that the magnitude of the transverse aerodynamic forces on airships of different fineness ratios are nearly proportional to the square of the diameter and almost independent of the length; whereas it has been generally assumed hitherto that these forces are more nearly proportional to the product of the length and the diameter. It follows also that the area of the tail surfaces should be approximately proportional to the square of the diameter rather than to the two-thirds power of the volume.

An exception is when the ratio length/diameter is small, less than about 5. In this case the factor $k_2 - k_1$ becomes of such importance that the aerodynamic

forces and the required size of the fins and control surfaces vary approximately as the volume to the two-thirds power.

TABLE 1.

COEFFICIENTS OF ADDITIONAL MASS FROM PROF. H. LAMB, R. & M. No. 623.

Length/diameter.	k_2	k_1	$k_2 - k_1$
1.00	0.5	0.5	0
1.50	.621	.305	.316
2.00	.702	.209	.493
2.51	.763	.156	.607
2.99	.803	.122	.681
3.99	.860	.082	.778
4.99	.895	.059	.836
6.01	.918	.045	.873
6.97	.933	.036	.897
8.01	.945	.029	.916
9.02	.954	.024	.930
9.97	.960	.021	.939
10.00	1.000	0	1.000

APPENDIX IV.

Application of Correction Factors to the Calculation of the Longitudinal Strength of Airship ZR.1

Tables 1 and 2 show calculations for the moment of inertia of the midship section of the ZR.1. It is assumed that all the longitudinals, including those in the keel, contribute their full cross-sectional areas to the moment of inertia. The calculations in Table 2 for the contribution of the shear wires are in accordance with Prof. Hovgaard's method of fictitious bars, except that the positions of the bars are taken to be midway between the two ends of the wires, instead of at the joints, and it is assumed that the initial tension is sufficient to make both shear and counter-shear wires effective.

From Tables 1 and 2, and multiplying by 2 for both sides of the hull:—

$$\Sigma A = 2\,(2481.6 + 155.6) = 5274 \text{ mm.}^2$$

$$\Sigma AH = 2\,(-4058 + 104.8) = 7906 \text{ m.} \times \text{mm.}^2.$$

$\Sigma AH / \Sigma A = -7906/5274 = -1.50$ m. = the distance of the neutral axis below the geometrical axis. The moment of inertia about the neutral axis is given by:—

$$I = 2\,(186{,}315 + 9{,}741) - (1.5^2 \times 5274)$$
$$= 380{,}220 \text{ m.}^2 \times \text{mm.}^2$$
$$= 590 \text{ m.}^2 \times \text{in.}^2.$$

Expressing the moment of inertia in terms of metres^2 × inches^2 may seem cumbersome; but it is really most convenient because of the practice of using metric units for the principal linear dimensions of the hull (*e.g.*, 5, 10 and 15 metre frame spacing), so that the bending moment is expressed in metre/lbs., while the stresses are desired in the customary units of lbs./in.^{2}.

Table 3 shows the unit stresses and total loads on the longitudinals in terms of the bending moment, M, the calculation being according to the bending theory with the fictitious bars included in the moment of inertia.

Table 4 shows the stresses in the shear and counter-shear wires, assuming both to be effective, in terms of the transverse shear, S, calculated by the method of shears. Table 5 shows the longitudinal shear in each main panel (*i.e.*, between consecutive main longitudinals) which the wires will take when their tensions are in accordance with the shear theory; and this table also shows the longitudinal shear to be taken in each panel, according to the bending theory. The longitudinal shear which the wires will take is obtained by multiplying the wire tensions in Table 4 by $\cos\theta$ and the number of wires in the panel. The longitudinal shear is calculated from the expression:—

$$Sl = (S\Sigma Ay)/I$$

where Sl = the longitudinal shear in the panel and ΣAy is summed over all longitudinals farther from the neutral axis than the panel under consideration. The expression is derived in the same way as the one for the tensions in the wires by the bending theory. In Table 5 it is assumed that $S = 1$ lb.

Comparison of the last two columns of Table 5 shows the discrepancies between the bending and the shear theories, or in other words, the discrepancies between the shears which the wires are required to take and what they can take, assuming the transverses to remain plane. Actually the transverses must distort until equilibrium is attained with stresses somewhere between the limits set by the two theories. From the magnitude of the discrepancy the correction factors for a revised application of the bending theory are to be obtained by judgment. In panels EG and GI the discrepancy is so small that it may be neglected and the correction factor for longitudinals E to I inclusive is 1.0. Because of the decrease in the sizes of the main shear wires above girder C, the longitudinal shear which the wires can take in panel CE, assuming their tensions to be determined by the transverse shear theory, is only 84.5 per cent. of the shear to be taken according to the bending theory. In order to be on the safe side in estimating the load taken by the longitudinals C and D, the correction factor will be taken as .95, or approximately one-third way from the bending theory towards the shear theory. In panel AC the capacity of the wires to take longitudinal shear is 65.5 per cent. of the shear to be taken, and a correction factor of .9 will be assumed.

In the lower part of the hull the discrepancies are much larger than in the upper part, owing to the combined effects of the reduced size of the main shear wires, the omission of the secondary shear wires, and the clustering of the longitudinal girders in the keel. In panel IK the capacity of the shear wires is 48.5 per cent. of the requirements of the bending theory, and a correction factor of .85 appears to be on the safe side. In panel KM the discrepancy is very large, showing the wires to be only 30.7 per cent. as large as required by the bending theory. On account of the poor shearing connection between the M and the other keel longitudinals, an average correction factor of .60 may be assumed for the girders below K; but the effect of the wiring of the keel (the MN panel) calls for special consideration. Because of the re-entrant angle of the MN panel, its wiring actually tends to reverse the stress in the N girder, that is, if the tensions in the wiring of the MN panel are in accordance with the shear theory, the N girder is in tension when the airship is in hogging. A view suggested by Prof. Hovgaard that the keel be regarded as a separate girder, but bending with the hull, would also show this state of affairs. It is difficult, however, to believe that the N girder can really be in tension when the lower part of the hull is in compression. Until experimental evidence is obtained regarding the end loads taken by the keel, it seems conservative to assign a correction factor of .3 to the N and P girders.

The walk-way girder, designated O, is not connected by wiring to the rest of the structure, but on account of its position, midway between the two M longitudinals, to which it is connected by short transverse girders at 2.5 metre

intervals, it should be at least 60 per cent. effective, and the correction factor .60 will be assigned to it. There remains a large load to be assigned to the M longitudinals. Assuming a correction factor of .60 for the keel as a whole and for the O girder, and .3 for the N and P girders, the factor for the M girder works out to be 1.09.

Table 6 shows the calculation of the moment of inertia with the actual area of each longitudinal replaced by an effective area equal to the actual area multiplied by the correction factor.

Taking account of the contribution of the wires to the longitudinal strength in the same manner as before, the moment of inertia of the midship section is found to be 490 m.^{2} × in.^{2}, with the neutral axis .44 m. below the geometrical axis. Using this corrected value of I, the unit stress in any longitudinal is given by

$$f = Mky/I$$

and the total end load $= Af$.

Table 7 gives the calculations of the unit stresses and total end loads in terms of the bending moment. Comparison of Tables 3 and 7 shows that the use of the correction factors makes almost no difference in the calculated stresses in the upper part of the hull. In the lower part the discrepancies between the shear and bending theories are so very large that estimates of the proper magnitude to assign to the correction factor may vary widely, especially for the N and P girders in the keel. Undoubtedly the actual loads taken by these girders will depend not only upon the bending moment at the section under consideration, but also upon the shear and bending for a considerable distance forward and aft of the section. It appears certain, however, that the contribution of the N and P girders to the longitudinal strength of the hull is small so that a possible error of 50 per cent. in the value ascribed to their correction factors will not be of much practical importance. The stress in the M longitudinal is found to be 44.5 per cent. greater with the correction factors than without them. This is a matter of some importance, but the M girder is particularly well supported at short intervals.

TABLE 1.

MOMENT OF INERTIA OF THE MIDSHIP SECTION OF THE AIRSHIP ZR.1, TAKING ACCOUNT OF ALL LONGITUDINAL GIRDERS, WITHOUT CORRECTION FACTORS.

Longitudinal.	Type No.	Area. A mm.^{2}	Height. H m.	H^2 m.^{2}	AH m. × mm.^{2}	AH^2 m.^{2} × mm.^{2}
A	56	108.0	11.82	139.71	1276	16082
B	21	136.1	11.53	132.94	1569	18093
C	54	185.9	10.39	107.95	1932	20068
D	21	136.1	8.74	76.39	1190	10397
E	54	185.9	6.44	41.47	1197	7079
F	21	136.1	3.83	14.67	521	1997
G	54	185.9	0.93	0.86	173	160
H	91	136.1	−2.02	4.08	−275	555
I	54	185.9	−4.81	23.14	−894	4302
J	91	136.1	−7.34	53.88	−999	7333
K	54	185.9	−9.38	87.98	−1744	16355
L	92	167.0	−10.83	117.29	−1810	19587
M	55	197.4	−11.67	136.19	−2303	26884
N	43C	104.8	−8.76	76.74	−918	8042
O	31A	72.1	−11.64	135.49	−840	9776
P	72	220.8	−9.66	93.32	−2133	20605
		2481.6			−4058	186315

TABLE 2.

CONTRIBUTION OF THE SHEAR WIRES TO THE MOMENT OF INERTIA OF THE MIDSHIP SECTION OF THE AIRSHIP ZR.1, CALCULATED BY PROF. HOVGAARD'S METHOD OF FICTITIOUS BARS.

MAIN SHEAR WIRES.

Panel.	$2a$ mm.^{2}	$2a.q.\cos^3\phi = a$ mm.^{2}	H m.	H^2 m.^{2}	aH mm.$^2 \times$ m.	aH^2 mm.$^2 \times$ m.^{2}
AC	10.54	19.2	11.53	132.9	222	2550
CE	10.54	19.2	8.74	76.4	168	1468
EG	13.18	24.0	3.83	14.67	92	352
GI	13.18	24.0	− 2.02	4.08	− 48.4	98
IK	13.18	24.0	− 7.34	53.9	− 176	1270
KM	10.54	19.2	− 10.83	117.3	− 208	2254
MN	13.18	5.6	− 10.21	104.2	− 57.4	586
		135.2			− 7.8	8578

SECONDARY SHEAR WIRES.

AC	6.64	5.10	11.53	132.9	58.8	678
CE	6.64	5.10	8.74	76.4	44.6	390
EG	6.64	5.10	3.83	14.67	19.5	74.7
GI	6.64	5.10	− 2.02	4.08	− 10.3	20.8
		20.4			112.6	1163.5
		135.2			− 7.8	8578
		155.6			104.8	9741

TABLE 3.

END LOAD ON THE LONGITUDINALS AT THE MIDSHIP SECTION OF AIRSHIP ZR.1, IN TERMS OF THE BENDING MOMENT, M, CALCULATED BY BEAM THEORY WITHOUT CORRECTION FACTORS.

Longitudinal.	Area. in.^{2}	Height from neutral axis. m.	Unit stress. M/(m. in.^{2})	Total load. M/m.
A	.333	13.32	.0226	.00752
B	.211	13.03	.0221	.00466
C	.287	11.80	.0201	.00576
D	.211	10.24	.0174	.00368
E	.287	7.94	.0134	.00384
F	.211	5.33	.0090	.00190
G	.287	2.43	.0041	.00118
H	.211	− 0.52	− .0009	− .00019
I	.287	− 3.31	− .0056	− .00161
J	.211	− 5.84	− .0099	− .00209
K	.287	− 7.88	− .0133	− .00382
L	.258	− 9.33	− .0158	− .00407
M	.305	− 10.17	− .0173	− .00527
N	.323	− 7.26	− .0123	− .00398
O	.222	− 10.14	− .0174	− .00386
P	.336	− 8.16	− .0138	− .00464

TABLE 4.

STRESSES IN THE SHEAR WIRES IN TERMS OF THE SHEARING FORCE, *S*, AT THE MIDSHIP SECTION OF THE ZR.1.

MAIN SHEAR WIRES.

Panel.	a in.^{2}	ϕ deg.	θ deg.	sin ϕ	cos ϕ	cos θ	Q in.^{2}	T *S.* lbs.
AC	.00817	30.6	75·75	·5075	.8607	.2462	.000110	.0154
CE	.00817	30.6	47·25	·5075	.8607	.6788	.000837	.0425
EG	.01021	30.6	18.75	·5075	.8607	.9469	.002035	.0741
GI	.01021	30.6	9.75	·5075	.8607	.9855	.002205	.0771
IK	.01021	30.6	38.25	·5075	.8607	.7854	.001400	.0614
KM	.00817	30.6	66.75	·5075	.8607	.3947	.000283	.0247
MN	.01021	58.0	32.7	.8480	.5299	.8415	.002765	.0677
							.009635	

SECONDARY SHEAR WIRES.

AC	.00515	49.8	75·75	·7638	.6455	.2462	.000118	.0110
CE	.00515	49.8	47·25	·7638	.6455	.6788	.000895	.0302
EG	.00515	49.8	18.75	·7638	.6455	.9469	.001740	.0421
GI	.00515	49.8	9.75	·7638	.6455	.9855	.001885	.0439
							.004638	
							.009635	

$$Q = a \,.\, \sin^2\phi \cos\phi \cos^2\theta$$

$$\Sigma a \,.\, \sin^2\phi \cos\phi \cos^2\theta = \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad .014273 \times 4$$

$$= \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad .057092 \text{ in.}^2$$

$$\text{Stress} = T = (a \,.\, \sin\phi \cos\phi \cos\theta / \Sigma a \sin^2\phi \cos\phi \cos^2\theta) \times S$$

TABLE 5.

COMPARISON OF LONGITUDINAL SHEAR IN EACH MAIN PANEL OF THE AIRSHIP ZR.1 ACCORDING TO THE SHEAR AND BENDING THEORIES. TOTAL TRANSVERSE SHEARING FORCE = 1 lb.

Panel.	Longitudinal shear from wires, assuming tensions are in accordance with shear theory. Main shear wires. lbs.	Secondary. shear wires. lbs.	Total. lbs.	Longitudinal shear required by bending theory. lbs.
AC	.053	.057	.110	.168
CE	.146	.156	·302	·357
EG	.255	.218	·473	·472
GI	.266	.226	·492	·508
IK	.211	—	.211	·435
KM	.085	—	.085	.277
MN	−·573	—	−·573	·133

TABLE 6.

CALCULATION FOR THE MOMENT OF INERTIA OF THE MIDSHIP SECTION OF THE ZR.1, USING ESTIMATED CORRECTION FACTORS.

Longitudinal.	Area. A mm.²	Correction Factor, k.	kA mm.²	Height H m.	kAH m.×mm.²	kAH^2 m.²×mm.²
A	108.0	.9	97.2	11.82	1150	13600
B	136.1	.9	122.5	11.53	1413	16300
C	185.9	.95	176.5	10.39	1839	19100
D	136.1	.95	129.5	8.74	1130	9880
E	185.9	1.0	185.9	6.44	1197	7080
F	136.1	1.0	136.1	3.83	521	2000
G	185.9	1.0	185.9	0.93	173	160
H	136.1	1.0	136.1	−2.02	−275	555
I	185.9	1.0	185.9	−4.81	−894	4300
J	136.1	.85	116	−7.34	−850	6230
K	185.9	.85	158	−9.38	−1482	13900
L	167.0	.60	100.2	−10.83	−1086	11750
M	197.4	1.09	215	−11.67	−2510	29300
N	104.8	.3	31.4	−8.76	−275	2410
O	72.1	.6	43.3	−11.64	−505	5870
P	220.8	.3	66.2	−9.66	−640	6200
			2085.7		−1094	148635

TABLE 7.

UNIT STRESS AND TOTAL END LOAD IN THE LONGITUDINALS AT THE MIDSHIP SECTION OF THE ZR.1 IN TERMS OF THE BENDING MOMENT, CALCULATED WITH CORRECTION FACTORS.

Longitudinal.	Area A in.²	Correction factor, k.	Height from neutral axis. m.	Unit stress $M/$(m. in²)	Total load $M/$m.
A	.333	.9	12.26	.0225	.00750
B	.211	.9	11.97	.0220	.00465
C	.287	.95	10.83	.0210	.00602
D	.211	.95	9.18	.0178	.00376
E	.287	1.0	6.88	.0141	.00405
F	.211	1.0	4.27	.0087	.00184
G	.287	1.0	1.37	.0028	.00080
H	.211	1.0	−1.58	−.0032	−.00068
I	.287	1.0	−4.37	−.0089	−.00256
J	.211	.85	−6.90	−.0120	−.00253
K	.287	.85	−8.94	−.0155	−.00445
L	.258	.60	−10.39	−.0127	−.00328
M	.305	1.09	−11.23	−.0250	−.00762
N	.323	.3	−8.32	−.0051	−.00165
O	.222	.6	−11.20	−.0137	−.00304
P	.336	.3	−9.22	−.00565	−.00190

APPENDIX V.

Stresses Due to Gas Pressure

In the parallel middle body of the airship ZR.1 the netting consists of steel wires running diagonally in two directions across the panels. The wires are 10.8 ft. long and 2.62 ft. apart, measured at right angles to their direction. Let p equal the mean gas pressure on the panel in lbs./ft.^{2}, then

$$P = .5 \times 10.8 \times 2.62 \times p = 14.15\, p.$$

Let h = the height of the centre of the panels above the bottom of the gas cell.
,, k = the unit lift of the gas.
,, p_0 = the gas pressure at the bottom of the cell.

Then $p = p_0 + kh$.

Given that $p_0 = 0.4''$ of water $= 2.1$ lbs./ft.^{2} and $k = .068$ lbs./ft.^{3}, then

$$p = 2.1 + .068\, h.$$

The wires are No. 13 gauge, $.072''$ dia., $a = .00407$ in.^{2} and $E = 30{,}000{,}000$ lbs./in.^{2}.

$$T = (Ea/24)^{1/3} \times P^{2/3} = 17.2\, P^{2/3} = 100.6\, p^{2/3}.$$

The wires of the netting are secured to the longitudinals in pairs at 3.28 ft. apart, inclined to the longitudinal axis of the girder at an angle of which the sine is 0.8. It follows that t, the tension in the netting per foot width (*i.e.*, per foot along the longitudinal), is given by $t = 2 \times .8 \times T/3.28 = .4875\, T = 49\, p^{2/3}$, and the radial load it throws upon the girder is $.4875\, T \cos \psi$, where ψ is the angle between the tangent to the netting at the girder and the radial plane (*i.e.*, the plane through the longitudinal axes of the girder and the hull).

In Fig. 20 of the section on the effect of gas pressure, c = the width of the panel $= 8.7$ ft., γ = the angle between the tangent to the netting at the girder, and the plane through the intersections of the netting with the longitudinal girders. Equilibrium requires that

$$\sin \gamma = pc/2t = 8.92\, p/T.$$

The apeces of the longitudinals all lie upon a circle; but owing to the secondary longitudinals being shallower than the main ones, their bases lie upon a circle concentric with, but larger than that passing through the bases of the main longitudinals. The lower secondary longitudinals are intermediate in depth between the main and the upper secondary longitudinals, and their bases lie on a circle between the other two.

Because of these differences in the distances of the bases of the main and secondary longitudinals from the centre of the cross-section, the netting is inclined at different angles to the radial planes at the main and secondary longitudinals. In any panel:—

Let λ_1 = the inclination of the plane through the bases of the longitudinals at the sides of the panel to the radial plane through the main longitudinal.
Let λ_2 = the corresponding angle at the secondary longitudinal.
Let ψ_1 = the inclination of the netting to the radial plane at the main longitudinal.
Let ψ_2 = the corresponding angle at the secondary longitudinal.

The angles ψ_1 and ψ_2 are measured exteriorly to the cross-section, as shown in Fig. 1, even when the interior angle is smaller.

$$\psi_1 = 180^\circ - \gamma - \lambda_1$$
$$\psi_2 = 180^\circ - \gamma - \lambda_2$$

γ is obtained from $\sin \gamma$, determined in the manner already described, and as shown in the sample calculation in Table 1. λ_1 and λ_2 are obtained from the drawings of the cross-section of the hull.

The total radial load on the longitudinal girder is the sum of the radial components of the netting tensions on each side of it, and the direct load of the gas pressure upon its base. In the ZR.1 the width of the base of the main longitudinals upon which the gas cell bears is 1.04 ft., and the corresponding width of the upper and lower secondary longitudinals are 0.66 and 0.82 ft. respectively.

The tangential load is wholly due to the netting and is equal to the difference of the tensions in the netting on the two sides of the longitudinal multiplied by $\sin\psi$. The value of $\sin\psi$ is so very close to 1.0 that it may be neglected, without appreciable error. Table 2 shows the radial and tangential loads corresponding to the conditions of gas pressure and netting tension calculated in Table 1.

Because of the symmetry of the loading on each side of the transverses, the 5-metre lengths of the longitudinals between the main and intermediate transverses are considered to be fixed-ended beams. A separate calculation is made for the effect of the movement of the intermediate transverse. The maximum stresses occur at the joints with the transverses, and the stresses due to the radial loads are calculated by the usual formula:—

$$f = wL^2/12\,Z,$$

where f = the maximum unit stress.
w = the running load.
L = the length of the girder.
Z = the section modulus.

It will now be shown that the tangential loads are modified by the support given by the netting against tangential deflection of the longitudinals.

The netting imposes gas pressure loads upon the longitudinals, both radially and tangentially, but it also supports them against tangential deflection. This support is so important that it must be taken into account in computing the stresses in the longitudinals due to the tangential load.

It may be seen from Fig. 1a, showing a typical panel of netting in the parallel middle body of the ZR.1, that the netting wires are secured to the longitudinals at approximately 2.83 ft. and 6.57 ft. from the transverses. The wires secured to the 2.83 ft. points are ineffective in resisting tangential deflection of the longitudinals because their other ends are secured to the next longitudinals as far or farther from the frames. Two of the wires secured at each 6.57 ft. point run to the joints between the next longitudinal and the transverses, so that they are effective against the tangential deflection of the longitudinal.

The tangential forces on any longitudinal girder may be regarded as composed of two systems of oppositely acting loads. One is the uniform tangential load calculated by the methods already described; and the other consists of forces concentrated at 6.57 ft. from each transverse, and of a magnitude depending on the tensions produced in the wires by the actual deflections at those points; and this deflection is the difference of the deflections which would be produced by each system acting alone.

Let y_u = the deflection at the 6.57 ft. points due to the uniform load.
,, y_c = the deflection at those points from the concentrated loads.
,, y_r = the total deflection at those points.

Then $y_r = y_u - y_c$.

By a well-known textbook formula for the deflection of a uniformly loaded fixed-ended beam

$$y_u = (w/12\,EI)\,(Lx^3 - x^4/2 - L^2x^2/2) \times 1728,$$

where w = the uniform tangential load in lbs./ft.
E = the modulus of elasticity of the girder in bending, assumed 9,000,000 lbs./in.^{2}.
I = moment of inertia of the girder about an axis radial to the hull.
L = length of the girder = 16.4 ft.
x = 6.57 ft.

Substituting these quantities, it is found that

$$y_u = .0334\, w/I.$$

By a formula for the deflections due to two symmetrically disposed concentrated loads

$$y_c = (F/EI)\,(x^3/3 - x^4/2L) \times 1728$$
$$= .00718\, F/I,$$

where F = the concentrated tangential loads at the 6.57 ft. points due to the tensions produced in the wires by the deflection y_r.

Also

$$F = (2 \times .8^2 \times Ey_r/L) = 1210\, y_r,$$
$$y_c = (.00718 \times 1210\, y_r/I) = 8.7\,(y_r/I)$$
$$y_r = y_u - 8.7\,(y_r/I) = .0334\, w/(I + 8.7)$$

Let M_u, M_c and M_r be the bending moments at the joints of the girders with the transverses due, respectively, to the uniform tangential load, the concentrated loads at the 6.57 ft. points, and the resultant of these two. The magnitude of these bending moments, and the maximum fibre stresses due to M_r in the conditions of the sample calculation of Table 1, are shown in Tables 2 and 3. In Fig. 23 the curves ABC and DEFG represent the bending moments of a main longitudinal due to the uniform and concentrated loads respectively. The difference between them, represented by the shaded area, is relatively small, showing how greatly the wires from the 6.57 ft. points reduce the magnitude of the tangential bending.

Up to this point in the analysis it is assumed that the longitudinals are *en castré* at both the main and intermediate transverses. The effect of the movement of the intermediate frame relatively to the neighbouring main frames will now be considered.

It is assumed that above longitudinal L the intermediate frame rises bodily without distortion. It is shown elsewhere in this paper that both theory and experiment justify this assumption. The lift of the gas upon the intermediate transverse is opposed by three sets of members as follows:—

(*a*) The shear wires.
(*b*) The longitudinals.
(*c*) The gas cell netting.

The resistance of the shear wires to the rise of the frame is calculated by the method of shears, which is discussed in the section on longitudinal strength. The force required to lift the frame 1 in. against the resistance of the wires is given by

$$F_s = (E/j)\,\Sigma a \sin^2\phi \cos\phi \cos^2\theta,$$

where F_s = the force required.
j = the distance between the main and secondary frames = 197 in.

The other symbols have the same meanings as in the method of shears for calculating the forces in the wires.

Table 4 shows the calculation of $\Sigma a \sin^2\phi \cos\phi \cos^2\theta$. The wires are considered to have sufficient initial tension to transmit compressive forces, so that

the summation is multiplied by 8 to include shear and counter-shear wires, both sides of the hull, and forward and aft of the intermediate frame.

$$F_s = (30{,}000{,}000 \times .092/197) = 14{,}000 \text{ lbs.}$$

The resistance of the longitudinals to 1 in. upward movement of the intermediate transverse may be calculated from the sum of the work done in bending all the longitudinals 1 in. upward midway between the main frames.

Let u = the angle between the longitudinal vertical plane and the plane determined by the longitudinal axes of a girder and of the hull.

When the vertical deflection of a girder is 1, the radial and tangential components of the deflection are $\cos u$ and $\sin u$ respectively. The vertical components of radial and tangential forces are, respectively, $\cos u$ and $\sin u$ times the force.

Let I_r and I_t be, respectively, the moments of inertia of a longitudinal girder in the radial and tangential directions.

The summations of $I_r \cos^2 u$ and $I_t \sin^2 u$ are shown in Table 5.

The force required to lift the intermediate transverse 1 in. against the resistance of the longitudinals, each 394 in. long, with fixed ends, is given by

$$F_e = 192\, E\, (\Sigma I_r \cos^2 u + \Sigma I_t \sin^2 u)/(394)^3$$
$$= 3{,}440 \text{ lbs.}$$

The resistance of the netting to upward movement of the transverse may be calculated in a manner analogous to the method of calculating the resistance presented by a fabric envelope to shear. (See Haas and Dietzius in the Third Annual Report of the American N.A.C.A.) The maximum shearing force is at mid-height of the hull and its magnitude is given by

$$S_{max} = F_n/2\pi R,$$

where S_{max} = the maximum shearing force per foot of transverse arc of the cross-section.

F_n = the force required to lift the intermediate frame 1 in. against the resistance of the netting.

R = the mean radius of the cross-section of the hull along the netting = 38.2 ft.

The tension in the wires of the netting due to the shear is given by

$$T = Sd/\sin\phi,$$

where d = the distance between the wires measured along the arc of the cross-section.

= 4.8 ft.

ϕ = the inclination of the netting wires to the longitudinal girders.

$$\sin\phi = 0.8, \quad \cos\phi = 0.6.$$
$$T_{max} = S_{max} \times 4.8/.8 = 6\, S_{max}.$$

Also $T_{max} = Ea \sin\phi \cos\phi/j$

$= 30{,}000{,}000 \times .00407 \times .8 \times .6/197$

$= 298$ lbs.

$S_{max} = 298/6 = 49.6$ lbs./ft.

$F_n = 2\pi R S_{max} = 2\pi \times 38.2 \times 49.6$

$= 11{,}900$ lbs.

The total force required to lift the intermediate transverse 1 in. relatively to the neighbouring main transverses is $F_s + F_e + F_n = 14{,}000 + 3{,}440 + 11{,}900$ = 29,340 lbs.

The lifting force is assumed to be one-half the gross lift of the gas cell minus half the weight of the structure above the keel between consecutive main frames. The average volume of the gas cells in the parallel middle body of the

ZR.1 is 148,000 cu. ft. The weight of the structure above the keel between main frames is about 1,960 lbs. It follows that the upward movement of the intermediate frame should be equal to

$$.5\,(148{,}000\,k - 1{,}960)/29{,}340$$

where k is the lift of the gas per unit volume. When the cell is filled with gas lifting .068 lbs./ft.^{3} the calculated rise of the frame is .138 in., and with gas lifting .063 lbs./ft.^{3} it is .126 in.

Table 6 shows the calculation of the stresses in the longitudinals due to the rise of intermediate transverse. The notation in this table is as follows:—

h_1 = the distance from the minor principal axis of the girder to the centre of gravity of the apex channel.
h_2 = the corresponding distance to the base channels.
h_3 = the distance from the major principal axis to the centre of gravity of the base channels.
u = the inclination of the major principal axis to the vertical.
f_a = the stress in the apex channels at the joints with the main frames, due to the rise of the intermediate frame.
f_b = the corresponding stress in the base channels.

Let Δ = the rise of the intermediate frame.

$$f_a = 24\,Eh_1\Delta/b^2.$$

When the cell is filled with gas lifting .068 lbs./ft.^{3}, $\Delta = .138$ in., as was shown above, and

$$f_a = \{\ 24 \times 9{,}000{,}000 \times .138/(394)^2\ \}\ h_1 \cos u$$
$$= 192\ h_1 \cos u.$$

Similarly,

$$f_b = 192\ h_2 \cos u \pm 192\ h_3 \sin u.$$

Table 7 shows the total stresses in the longitudinals due to the gas pressure and the lift of the intermediate transverse.

TABLE 1.

GAS PRESSURE ON PANELS AND TENSIONS OF NETTING IN PARALLEL MIDDLE BODY OF AIRSHIP ZR.1, WITH GAS LIFTING .068 lbs./ft.^{3} AND 2.1 lbs./ft.^{2} SUPER-PRESSURE.

Panel.	AB	BC	CD	DE	EF	FG
Height, h ft.	77·5	75·0	70·5	63·8	55·7	46·7
Unit pressure, p lbs./ft.^{2} ...	7·37	7·20	6·90	6·44	5·89	5·28
Load on wire, P lbs. ...	104.2	102.0	97.6	91.1	83.4	74.8
$(Ea/24)^{1/3}$	17.2	17.2	17.2	17.2	17.2	17.2
$P^{2/3}$	22.1	21.8	21.2	20.2	19.05	17.75
Tension, T lbs.	380	375	365	348	328	305
$\sin\gamma$	.1730	.1715	.1688	.1650	.1603	.1545
γ	9° 58′	9° 53′	9° 45′	9° 30′	9° 13′	8° 53′
λ_1	85° 39′	85° 39′	85° 39′	85° 39′	85° 39′	85° 39′
λ_2	80° 6′	80° 6′	80° 6′	80° 6′	80° 6′	80° 6′
$\psi_1 = 180° - \gamma - \lambda_1$	84° 23′	84° 28′	84° 36′	84° 51′	85° 8′	85° 28′
$\psi_2 = 180° - \gamma - \lambda_2$	89° 56′	90° 1′	90° 9′	90° 24′	90° 41′	91° 1′
$\cos\psi_1$	.0979	.0964	.0941	.0898	.0848	.0791
$\cos\psi_2$	.0011	−.0003	−.0026	−.0070	−.0119	.0178
$.4875\ T\cos\psi_1$	18.15	17.65	16.75	15.27	13.58	11.8
$.4875\ T\cos\psi_2$	0.21	−.05	−.46	−1.19	−1.90	−2.65

Panel.	GH	HI	IJ	JK	KL	LM
Height, h ft.	37.0	27.6	18.8	11.15	5.31	1.57
Unit pressure, p lbs./ft.^{2} ...	4.62	3.98	3.38	2.86	2.46	2.21
Load on wire, P lbs. ...	65.4	56.3	47.8	40.5	34.8	31.3
$(Ea/24)^{1/3}$	17.2	17.2	17.2	17.2	17.2	17.2
$P^{2/3}$	16.2	14.65	13.15	11.8	10.62	9.90
Tension, T lbs.	279	252	226	203	183	170
$\sin\gamma$	.1480	.1410	.1335	.1258	.1200	.1159
γ	8° 31′	8° 6′	7° 40′	7° 14′	6° 54′	6° 39′
λ_1	84° 26′	84° 26′	84° 26′	84° 26′	84° 26′	84° 26′
λ_2	81° 19′	81° 19′	81° 19′	81° 19′	81° 19′	81° 19′
$\psi_1 = 180° - \gamma - \lambda_1$	87° 3′	87° 28′	87° 54′	88° 20′	88° 40′	88° 55′
$\psi_2 = 180° - \gamma - \lambda_2$	90° 10′	90° 35′	91° 1′	91° 27′	91° 47′	92° 2′
$\cos\psi_1$	.0515	.0442	.0366	.0291	.0233	.0189
$\cos\psi_2$	−.0029	−.0102	−.0178	−.0253	−.0311	−.0355
$.4875\ T\cos\psi_1$	7.0	5.43	4.03	2.88	2.09	1.57
$.4875\ T\cos\psi_2$	−.39	−1.25	−1.96	−2.50	−2.78	−2.95

TABLE 2.

GAS PRESSURE LOADS AND STRESSES IN MAIN LONGITUDINALS OF AIRSHIP ZR.1 WITH GAS LIFTING .068 lbs./ft.^{3} & 2.1 lbs./ft.^{2} SUPER-PRESSURE.

Longitudinal.	A	C	E	G	I	K	M
Radial load from netting, lbs./ft. ...	36.30	34.40	28.85	18.80	9.46	4.96	1.57
Unit pressure, p lbs./ft.^{2}	7.37	7.05	6.16	4.95	3.68	2.66	2.21
Direct radial load, lbs./ft.	7.65	7.33	6.41	5.15	3.83	2.76	2.30
Total radial load, lbs./ft.	43.95	41.73	35.26	23.95	13.29	7.72	3.67
$I/y=Z$ in.^{3}	1.725	1.46	1.46	1.46	1.46	1.46	1.74
$g=wL^2/Z$ lbs./in.^{2}	−6850	−7690	−6500	−4410	−2445	−1420	−598
Tangential load, lbs./ft.	0	4.88	9.75	12.68	12.68	9.75	83.0
I (tangentially) in.^{4}	—	5.05	5.05	5.05	5.05	5.05	—
$y_u=.0334\ w/I$ in.	0	.0323	.0645	.0838	.0838	.0645	—
$y_r=.0334\ w/(I+8.7)$	0	.0119	.0237	.0308	.0308	.0237	—
$F=1210\ y_r$ lbs.	0	14.4	28.7	37.3	37.3	28.7	—
$M_u=wL^2/12$ ft./lbs.	0	109	218	284	284	218	—
$M_c=3.94\ F$ ft./lbs.	0	57	113	147	147	113	—
$M_r=M_u-M_c$ ft./lbs.	0	52	105	137	137	105	—
Z (tangentially) in.^{3}	—	.950	.950	.950	.950	.950	—
$f=12\ M_r/Z$ lbs./in.^{2}	0	656	1327	1730	1730	1327	—

TABLE 3.

GAS PRESSURE LOADS AND STRESSES IN SECONDARY LONGITUDINALS OF AIRSHIP ZR.1 WITH GAS LIFTING .068 lbs./ft.^{3} & 2.1 lbs./ft.^{2} SUPER-PRESSURE.

Longitudinal.	B	D	F	H	J	L
Radial load from netting, lbs./ft. ...	0.16	−1.65	−4.55	−1.64	−4.46	−5.78
Unit pressure, p lbs./ft.^{2}	7.29	6.67	5.58	4.30	3.12	2.33
Direct radial load, lbs./ft.	4.81	4.40	3.69	3.52	2.56	1.91
Total radial load, lbs./ft.	4.97	2.75	−0.86	1.88	−1.90	−3.87
$I/y=Z$ in.^{3}	.656	.656	.656	.802	.802	.960
$f=wL^2/Z$ lbs./in.^{2}	−2040	−1128	352	−630	637	1085
Tangential load, lbs./ft.	2.44	8.30	11.2	13.15	11.2	6.34
I (tangentially) in.^{4}	1.11	1.11	1.11	1.85	1.85	2.31
$Y_u=.0334\ w/I$ in.	.0735	.250	.337	.2375	.202	.0916
$Y_r=.0334\ w/(I+8.7)$	.0083	.0282	.0382	.0415	.0354	.0192
$F=1210\ y_r$ lbs.	10.1	34.1	46.2	50.2	42.8	23.2
$M_u=wL^2/12$ ft./lbs.	54.6	186	251	294	251	142
$M_c=3.94\ F$ ft./lbs.	39.8	134.3	182	198	169	91.4
$M_r=M_u-M_c$ ft./lbs.	14.8	51.7	69	96	82	50.6
Z (tangentially) in.^{3}	.364	.364	.364	.470	.470	.587
$f=12\ M_r/Z$ lbs./in.^{2}	488	1710	2280	2450	2100	1033

TABLE 4.

RESISTANCE OF THE SHEAR WIRES TO THE LIFT OF THE INTERMEDIATE TRANSVERSES IN THE PARALLEL MIDDLE BODY OF AIRSHIP ZR.1

MAIN SHEAR WIRES.

Panel.	a in.^{2}	ϕ deg.	θ deg.	sin ϕ	cos ϕ	cos θ	$a \sin^2 \phi \cos \phi \cos^2 \theta$
AC	.00817	30.6	75.75	.5075	.8607	.2462	.000110
CE	.00817	30.6	47.25	.5075	.8607	.6788	.000837
EG	.01021	30.6	18.75	.5075	.8607	.9469	.002035
GI	.01021	30.6	9.75	.5075	.8607	.9855	.002205
IK	.01021	30.6	38.25	.5075	.8607	.7854	.001400
KM	.00817	30.6	66.75	.5075	.8607	.3947	.000283
	.05514						.006870

SECONDARY SHEAR WIRES.

AC	.00515	49.8	75.75	.7638	.6455	.2462	.000118
CE	.00515	49.8	47.25	.7638	.6455	.6788	.000895
EG	.00515	49.8	18.75	.7638	.6455	.9469	.001740
GI	.00515	49.8	9.75	.7638	.6455	.9855	.001885
	.02060						.004638
	.05514						.006870
	.07574						

$$\Sigma a \cdot \sin^2 \phi \cos \phi \cos^2 \theta = .011508 \times 8$$
$$= .092 \text{ in.}^2$$

TABLE 5.

RESISTANCE OF THE LONGITUDINALS TO THE LIFT OF THE INTERMEDIATE TRANSVERSES IN THE PARALLEL MIDDLE BODY OF AIRSHIP ZR.1.

Longitudinal.	Type.	I_r in.^{4}	I_t in.^{4}	u deg.	cos u.	sin u.	$I_r \cos^2 u$. in.^{4}	$I_t \sin^2 u$. in.^{4}
A	56	7.2	—	0	1.0	0	7.20	0
B	21	2.6	1.11	14.25	.9692	.2461	2.44	0.07
C	54	12.4	5.05	28.5	.8788	.4772	9.57	1.15
D	21	2.6	1.11	42.75	.7343	.6788	1.40	0.51
E	54	12.4	5.05	57.0	.5446	.8387	3.68	3.65
F	21	2.6	1.11	71.25	.3215	.9469	0.27	1.00
G	54	12.4	5.05	85.5	.0785	.9969	0.07	5.02
H	91	3.96	1.85	80.25	.1693	.9855	0.11	1.80
I	54	12.4	5.05	66.0	.4067	.9135	2.05	4.21
J	91	3.96	1.85	51.75	.6191	.7854	1.52	1.14
K	54	12.4	5.05	37.5	.7934	.6088	7.80	1.87
L	92	4.83	2.31	23.25	.9188	.3947	4.07	0.38
							40.18	20.78
							× 2	× 2
							80.36	41.56

TABLE 6.

STRESSES IN THE LONGITUDINALS OF THE AIRSHIP ZR.1 DUE TO THE LIFT OF THE INTERMEDIATE TRANSVERSE. CELL FILLED WITH GAS LIFTING .068 lbs./ft.^{3}.

Longitudinal.	cos u.	sin u.	h_1 in.	h_2 in.	h_3 in.	$f_a = 192\, h_1 \cos u$, lbs./in.^{2}	$f_b = 192\,(h_2 \cos u + h_3 \sin u)$ lbs./in.^{2}
A	1.0	0	8.35	5.16	5.31	−1605	996
B	.9692	.2461	3.97	3.04	3.05	−740	566 ± 144
C	.8788	.4772	8.30	5.21	5.31	−1400	880 ± 487
D	.7343	.6788	3.97	3.04	3.05	−560	428 ± 398
E	.5446	.8387	8.30	5.21	5.31	−868	545 ± 855
F	.3215	.9469	3.97	3.04	3.05	−245	188 ± 555
G	.0785	.9969	8.30	5.21	5.31	−125	78 ±1018
H	.1693	.9855	4.96	3.78	3.94	161	−123 ± 746
I	.4067	.9135	8.30	5.21	5.31	650	−407 ± 931
J	.6191	.7854	4.96	3.78	3.94	590	−450 ± 595
K	.7934	.6088	8.30	5.21	5.31	1265	−795 ± 621
L	.9188	.3947	5.04	3.70	3.94	890	−653 ± 298

TABLE 7.

TOTAL STRESSES IN THE LONGITUDINALS OF THE ZR.1 DUE TO THE GAS PRESSURE AND THE LIFT OF THE INTERMEDIATE TRANSVERSE. CELL FILLED WITH GAS AND 2.1 lbs./ft.^{2} SUPER-PRESSURE.

	Gas Lifting .068 lbs./ft.^{3}			Gas Lifting .057 lbs./ft.^{3}		
Longitudinal.	Apex channel. lbs./in.^{2}	Upper base channel. lbs./in.^{2}	Lower base channel. lbs./in.^{2}	Apex channel. lbs./in.^{2}	Upper base channel. lbs./in.^{2}	Lower base channel. lbs./in.^{2}
A	−8455	5225	5225	−7175	4485	4485
B	−2780	1498	2762	−1380	453	1659
C	−9090	4557	6843	−7700	3853	5827
D	−1688	−820	3400	−440	−1394	2066
E	−7368	2448	6812	6315	2086	5854
F	107	−2917	2753	890	−3205	1845
G	−4535	102	5598	−3940	120	4830
H	−469	−2838	3554	−83	−2324	2450
I	−1795	−1533	3789	−1660	−1243	3327
J	1227	−3630	1760	1258	−2965	1045
K	−155	−1851	2045	−305	−1305	1689
L	1975	−2782	−120	1825	−2315	−369

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